Prof. UAM dr hab. Wojciech Dybalski AMU Poznań
Winter semester 2025/26

Stochastic Differential Equations
Homework Sheet 4

Problem 1. Let $X, Y : \Omega \to \mathbb{R}$ be two independent random variables. Show that the characteristic functions satisfy

$$\phi_{X+Y}(u) = \phi_X(u)\phi_Y(u). \tag{1}$$

Now suppose that $P(X \in F) = \int_F p_X(x)dx$ and $P(Y \in F) = \int_F p_Y(y)dy$ for $p_X, p_Y \in S(\mathbb{R})$, i.e. X, Y have Schwartz class densities p_X, p_Y. Show that

$$p_{X+Y}(z) = \int p_X(z-y)p_Y(y)dy. \tag{2}$$

Problem 2. Let $\{B_t\}_{t\in\mathbb{R}_+}$ be the d-dimensional Brownian motion starting at zero and $v \in \mathbb{R}^d$ a fixed non-zero vector. Compute the distribution of $X_t^{(v)} := \langle v, (B_{t+1} - B_t)\rangle$.

To be discussed in class: 07.11.2025 (Together with the last two problems of HS3).