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AMU Poznań

Winter semester 2025/26

Stochastic Differential Equations
Homework Sheet 5

Problem 1. Let $\{B_t\}_{t\in\mathbb{R}_+}$ be a one-dimensional Brownian motion starting at $x \in \mathbb{R}$ and let $c > 0$ be a constant. Prove that the process

$$\tilde{B}_t := \frac{1}{c} B_{c^2 t} \tag{1}$$

is also a Brownian motion. Hint: Show that the finite-dimensional distributions of $\{B_t\}_{t\in\mathbb{R}_+}$ and $\{\tilde{B}_t\}_{t\in\mathbb{R}_+}$ coincide.

Problem 2. Show that the function

$$g(s) = \begin{cases} s\sin\left(\frac{1}{s}\right), & s \neq 0, \\ 0, & s = 0. \end{cases} \tag{2}$$

has infinite total variation on $[0, 1]$.

Problem 3. Suppose that $g \in C^1(\mathbb{R})$. Show that $V_a^b(g) = \int_a^b |g'(s)|ds$.
Hint: Consider separately the inequalities $V_a^b(g) \leq \int_a^b |g'(s)|ds$ and $V_a^b(g) \geq \int_a^b |g'(s)|ds$.

To be discussed in class: 13.11.2025