Quantum states for a scalar field on AdS

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0. Introduction

- Study systematically a massive scalar field on anti-de Sitter (AdS), namely the role of the boundary conditions and the existence of extensions of the solutions to the boundary as distributions.
- Construct quantum states holographically from the boundary of AdS, analogously with the asymptotically flat case [Dappiaggi et al], with the view of extending the procedure to asymptotically AdS spacetimes.
- Use this system as an example to study QFT on manifolds with boundaries and theories with singular potentials.

1 Massless, conformally coupled scalar field on AdS

2 Massive scalar field on AdS

3 Conclusions

Outline

1 Massless, conformally coupled scalar field on AdS

2 Massive scalar field on AdS

3 Conclusions

1.1. Poincaré fundamental domain.

- *Anti-de Sitter* AdS_{d+1} $(d \geq 2)$ is the maximally symmetric solution to Einstein's equations with a negative cosmological constant. It is defined by the relation

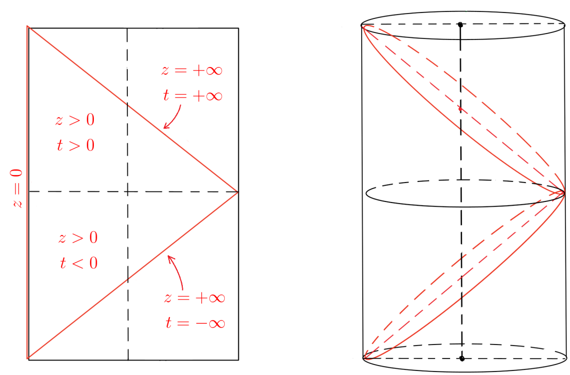
$$-X_0^2 - X_1^2 + \sum_{i=2}^{d+1} X_i^2 = -\ell^2\,, \qquad \ell > 0\,,$$

where $(X_0, \dots, X_{d+1})$ are Cartesian coordinates of $\mathbb{M}^{2,d}$.

- Poincaré patch (t, z, x_i), $t \in \mathbb{R}$, $z \in \mathbb{R}_{>0}$ and $x_i \in \mathbb{R}$, $i = 1, \dots, d-1$,

$$\mathrm{d}s^2 = \frac{\ell^2}{z^2}\left(-\mathrm{d}t^2 + \mathrm{d}z^2 + \delta^{ij}\mathrm{d}x_i\mathrm{d}x_j\right)\,.$$

The region covered by this chart is the *Poincaré fundamental domain*, PAdS_{d+1}.

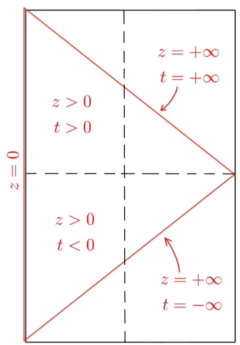


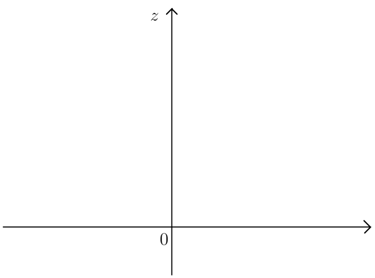
1.1. Poincaré fundamental domain.

- PAdS_{d+1} can be mapped to $\mathring{\mathbb{H}}^{d+1} \doteq \mathbb{R}_{>0} \times \mathbb{R}^d \subset \mathbb{M}^{1,d}$ via a conformal rescaling

$$\mathrm{d}s^2 \mapsto \frac{z^2}{\ell^2}\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}z^2 + \delta^{ij}\mathrm{d}x_i\mathrm{d}x_j \,.$$

  We can attach a conformal boundary as the locus $z = 0$ and obtain $\mathbb{H}^{d+1} \doteq \mathbb{R}_{\geq 0} \times \mathbb{R}^d$, the *upper half-plane*. This is the geometric setting of the Casimir-Polder system.





1.2. Massless, conformally coupled scalar field

- Poincaré domain (PAdS_{d+1}, g), scalar field $\phi : \mathrm{PAdS}_{d+1} \to \mathbb{R}$,

$$P\phi = \left(\Box + \frac{d-1}{4d} R\right) \phi = 0\,.$$

For every solution $\phi \in C^\infty(\mathrm{PAdS}_{d+1})$, the function $\Phi = (\frac{z}{\ell})^{\frac{1-d}{2}} \phi$ is a smooth solution of the same equation in $\mathring{\mathbb{H}}^{d+1}$.

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- Upper half-plane $(\mathbb{H}^{d+1}, \eta)$, scalar field $\Phi : \mathbb{H}^{d+1} \to \mathbb{R}$,

$$\begin{cases} P_{\mathbb{H}}\Phi = \Box_{\mathbb{H}}\Phi = 0 \\ \Phi(0, x_i) = 0 \end{cases},$$

  with Dirichlet boundary conditions at $\partial\mathbb{H}^{d+1}$ (Casimir-Polder system).

  The classical and quantum scalar field theory in $\mathbb{H}^{d+1}$ was investigated in [Dappiaggi, Nosari, Pinamonti (2014)].

1.2. Massless, conformally coupled scalar field

Off-shell configurations. Define the isometry $\iota_z : \mathbb{R}^{d+1} \to \mathbb{R}^{d+1}$, $(z, x^i) \mapsto (-z, x^i)$, and the natural action on functions, $f(x) \mapsto f(\iota_z(x))$.

The space of kinematical or off-shell configurations of the CP system is

$$\mathscr{C}^{\mathrm{CP}}(\mathbb{H}^{d+1}) \doteq \left\{ \phi \in C^\infty(\mathbb{H}^{d+1}) : \phi|_{\partial\mathbb{H}^{d+1}} = 0\,,\ \exists\, \tilde{\phi} \in C^\infty(\mathbb{R}^{d+1}) : \phi = \frac{1}{\sqrt{2}} \left(\tilde{\phi} - \iota_z \tilde{\phi} \right)\Big|_{\mathbb{H}^{d+1}} \right\}.$$

Classical observables. Functionals $F_f : \mathscr{C}^{\mathrm{CP}}(\mathbb{H}^{d+1}) \to \mathbb{R}$,

$$\phi \mapsto F_f(\phi) = \int_{\mathbb{H}^{d+1}} \mathrm{d}^{d+1}x\, \phi(x) f(x)\,, \qquad f \in C_0^\infty(\mathbb{H}^{d+1})\,.$$

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Off-shell $*$-algebra of observables. $\mathscr{A}^{\mathrm{CP}}_{\mathrm{off}}(\mathbb{H}^{d+1})$, endowed with complex conjugation as the $*$-operation, is the algebra generated by $F_f(\phi)$.

On-shell $*$-algebra of observables. $\mathscr{A}^{\mathrm{CP}}_{\mathrm{on}}(\mathbb{H}^{d+1})$ is the algebra generated by $F_{[f]}(\phi)$,

$[f] \in \mathscr{C}^{\mathrm{CP}}_0(\mathbb{H}^{d+1}) / P_{\mathbb{H}} \left[\mathscr{C}^{\mathrm{CP}}_0(\mathbb{H}^{d+1}) \right]$,

$\phi \in \mathscr{S}^{\mathrm{CP}}(\mathbb{H}^{d+1})$, the space of solutions of the CP system.

1.2. Massless, conformally coupled scalar field

States. $\omega : \mathscr{A}^{\mathrm{CP}}(\mathbb{H}^{d+1}) \to \mathbb{C}$ s.t. $\omega(1) = 1$ and $\omega(a^* a) \geq 0$, for all $a \in \mathscr{A}^{\mathrm{CP}}(\mathbb{H}^{d+1})$.

- States $\tilde{\omega}$ on $\mathscr{A}^{\mathrm{KG}}(\mathbb{R}^{d+1})$ for the Klein-Gordon system in Minkowski spacetime.

  Guassian or *quasi-free states*: the n-point functions $\tilde{\omega}_n \in \mathcal{D}'((\mathbb{R}^{d+1})^{\times n})$ are such that

$$\tilde{\omega}_{2n+1}(f_1 \otimes \ldots \otimes f_n) = 0\,,$$
$$\tilde{\omega}_{2n}(f_1 \otimes \ldots \otimes f_n) = \sum_{\pi_{2n} \in S'_{2n}} \prod_{i=0}^{n} \tilde{\omega}_2(f_{\pi_{2n}(i-1)} \otimes f_{\pi_{2n}(i)})\,,$$

  where S'_{2n} is the set of ordered permutations of $2n$-elements.

  Hadamard states: $\tilde{\omega}_2 \in \mathcal{D}'(\mathbb{R}^{d+1} \times \mathbb{R}^{d+1})$ has a wavefront set of the form

$$WF(\tilde{\omega}_2) = \left\{(x, x', k_x, -k_{x'}) \in T^*(\mathbb{R}^{d+1} \times \mathbb{R}^{d+1}) \setminus \{0\} : (x, k_x) \sim (x', k_{x'}),\ k_x \rhd 0\right\},$$

  $(x, k_x) \sim (x', k_{x'})$: x is linked to x' by a null geodesic,
  $k_{x'}$ is the parallel transport of k_x along that geodesic;
  $k_x \rhd 0$: k_x is future-directed.

1.2. Massless, conformally coupled scalar field

- States ω on $\mathscr{A}^{\mathrm{CP}}_{\mathrm{on}}(\mathbb{H}^{d+1})$ for the CP system.

  Hadamard states: requires a modified definition, inspired by the concept of *F-locality* of [Kay (1992)].

  Definition: A quasi-free state ω on $\mathscr{A}^{\mathrm{CP}}(\mathbb{H}^{d+1})$ is a *Hadamard state* if its restriction to any globally hyperbolic subregion of $\mathbb{H}^{d+1}$ is of Hadamard form.

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  Remarks:

  (1) The CP algebra $\mathscr{A}^{\mathrm{CP}}(\mathbb{H}^{d+1})$ is injectively embedded in the KG algebra $\mathscr{A}^{\mathrm{KG}}(\mathbb{R}^{d+1})$, hence a Hadamard state on $\mathscr{A}^{\mathrm{KG}}_{\mathrm{on}}(\mathbb{R}^{d+1})$ for the KG system in Minkowski spacetime can be pulled back to a state on $\mathscr{A}^{\mathrm{CP}}_{\mathrm{on}}(\mathbb{H}^{d+1})$ for the CP system, preserving the Hadamard property.

  (2) (Method of images) If $\tilde{\omega}_2 \in \mathcal{D}'(\mathbb{R}^{d+1} \times \mathbb{R}^{d+1})$ has an integral kernel such that

  $$\tilde{\omega}_2(z, x^i; z', x'^i) = \tilde{\omega}_2(-z, x^i; -z', x'^i)\,,$$

  then the corresponding $\omega_2 \in \mathcal{D}'(\mathbb{H}^{d+1} \times \mathbb{H}^{d+1})$ is such that

  $$\omega_2(z, x^i; z', x'^i) = \tilde{\omega}_2(z, x^i; z', x'^i) - \tilde{\omega}_2(-z, x^i; z', x'^i)\,.$$

1.2. Massless, conformally coupled scalar field

- States ω^{PAdS} on $\mathscr{A}^{\mathrm{KG}}_{\mathrm{on}}(\mathrm{PAdS}_{d+1})$ for the KG system in PAdS_{d+1}.

  Given a Hadamard, quasi-free state ω on $\mathscr{A}^{\mathrm{CP}}_{\mathrm{on}}(\mathbb{H}^{d+1})$, then a Hadamard, quasi-free state ω^{PAdS} on $\mathscr{A}^{\mathrm{KG}}_{\mathrm{on}}(\mathrm{PAdS}^{d+1})$ is such that the integral kernel of its two-point function is

$$\omega_2^{\mathrm{PAdS}}(z, x^i; z', x'^i) = \Omega^{\frac{d-1}{2}}(z)\, \omega_2(z, x^i; z', x'^i)\, \Omega^{\frac{d-1}{2}}(z')\,,$$

  where $\Omega(z) = \frac{z}{\ell}$ is the conformal factor.

Outline

1 Massless, conformally coupled scalar field on AdS

2 Massive scalar field on AdS

3 Conclusions

2. Massive scalar field on AdS

2.1. Field equation

- Poincaré domain (PAdS_4, g), scalar field $\phi : \mathrm{PAdS}_4 \to \mathbb{R}$,

$$P\phi = \left(\Box - \widetilde{m}^2 - \xi R\right)\phi = 0\,.$$

- Upper half-plane $(\mathbb{H}^4, \eta)$, scalar field $\Phi = (\frac{z}{\ell})^{-1}\phi : \mathring{\mathbb{H}}^4 \to \mathbb{R}$,

$$P_{\mathbb{H}}\Phi = \left(\Box_{\mathbb{H}} - \frac{m^2}{z^2}\right)\Phi = 0\,,$$

with $m^2 \doteq \tilde{m}^2 - (\xi - \frac{1}{6})R \geq 0$.

The KG system in PAdS_4 is equivalent to a scalar field system in $\mathbb{H}^4$ with a singular potential at $z = 0$.

2.2. Distributional solutions

$$P_{\mathbb{H}}\Phi = \left(\Box_{\mathbb{H}} - \frac{m^2}{z^2}\right)\Phi = 0$$

Case $m^2 = 0$. The field equation, and hence the smooth solutions, can be trivially extended to $z = 0$ to the whole of $\mathbb{M}^{1,3}$, regardless of the boundary conditions.

Case $m^2 > 0$. There are no smooth extensions of solutions to $z = 0$. Instead, consider distributional solutions in $\mathbb{H}^4$ for $m^2 > 0$ in three steps:

(1) Construct distributional solutions in $\mathring{\mathbb{H}}^4$;

(2) Show they admit an extension to $z = 0$ using the notion of scaling limit;

(3) Exploit the $z \to -z$ symmetry to extend the solutions to $\mathbb{M}^{1,3}$.

2.2. Distributional solutions

Step 1. Fourier representation of Φ:

$$\Phi = \int_{\mathbb{R}^3} \frac{d^3\underline{k}}{(2\pi)^{\frac{3}{2}}} e^{i\underline{k}\cdot\underline{x}} \widehat{\Phi}_{\underline{k}},$$

where $\widehat{\Phi}_{\underline{k}}$ are solutions of the ODE

$$Q\,\widehat{\Phi}_{\underline{k}} \doteq \left[\frac{d^2}{dz^2} + \left(\omega^2 - k_x^2 - k_y^2 - \frac{m^2}{z^2}\right)\right]\widehat{\Phi}_{\underline{k}} = 0\,.$$

Generic solution:

$$\widehat{\Phi}_{\underline{k}}(z) = c_1(\underline{k})\sqrt{z}\,J_\nu\left(z\sqrt{-\underline{k}\cdot\underline{k}}\right) + c_2(\underline{k})\sqrt{z}\,Y_\nu\left(z\sqrt{-\underline{k}\cdot\underline{k}}\right),$$

$$\nu \doteq \frac{1}{2}\sqrt{1+4m^2}\,.$$

Solution is well-defined for $z > 0$ and $z < 0$ (with $z \to -z$).

Remark: Imposing Dirichlet boundary conditions implies $c_2(\underline{k}) = 0$.

2.2. Distributional solutions

Step 2. $\widehat{\Phi}_{\underline{k}}(z) \in \mathcal{D}'(\mathbb{R} \setminus \{0\})$. Is there an extension as a distribution at the origin and, if so, is it still a solution of the equation?

We use the notion of scaling limit by [Steinmann (1971)] and the results of [Brunetti, Fredenhagen (2000)] and [Bahns, Wrochna (2014)].

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Definition: For a distribution $u \in \mathcal{D}'(\mathbb{R})$, let $u_\lambda \in \mathcal{D}'(\mathbb{R})$, $\lambda > 0$, be such that

$$u_\lambda(f) \doteq \lambda^{-1} u(f(\lambda^{-1} \cdot)) \,, \qquad f \in \mathcal{D}(\mathbb{R}) \,.$$

The *scaling degree* of u, $\mathrm{sd}(u)$, is the infimum over all $\alpha \in \mathbb{R}$ such that

$$\lim_{\lambda \to 0} \lambda^\alpha u_\lambda = 0 \,.$$

Theorem: Let $u \in \mathcal{D}'(\mathbb{R} \setminus \{0\})$ have scaling degree $\mathrm{sd}(u)$. Then, if $\mathrm{sd}(u) < 1$, u admits a unique extension to $\mathcal{D}'(\mathbb{R})$ with the same scaling degree, otherwise, if $\mathrm{sd}(u) \geq 1$, u admits several extensions to $\mathcal{D}'(\mathbb{R})$ with the same scaling degree.

2.2. Distributional solutions

Corollary: Let $u_1, u_2 \in C^\infty(\mathbb{R} \setminus \{0\})$ be respectively defined so that, for $x > 0$,

$$u_1(x) = \sqrt{x}\, J_\nu(x)\,, \qquad u_2(x) = \sqrt{x}\, Y_\nu(x)\,,$$

and, for $x < 0$, $u_1(x) = u_1(-x)$ and $u_2(x) = u_2(-x)$. Then, u_1 admits a unique extension to a distribution $u_{1,0} \in \mathcal{D}'(\mathbb{R})$ with the same scaling degree of u_1 for all $m^2 > 0$, whereas u_2 admits multiple extensions to a distribution $u_{2,0} \in \mathcal{D}'(\mathbb{R})$ with the same scaling degree of u_2 for $m^2 \geq 2$, and a unique extension for $0 < m^2 < 2$.

Are these extensions still solutions of the field equation?

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Are these extensions still solutions of the field equation?

Proposition: Let

$$\tilde{Q} \doteq z^2 Q = z^2 \frac{d^2}{dz^2} + z^2 \left(\omega^2 - k_x^2 - k_y^2\right) - m^2\,,$$

and $u \in \mathcal{D}'(\mathbb{R} \setminus \{0\})$ s.t. $\tilde{Q}u = 0$. Then, u admits an *on-shell extension* to $\mathcal{D}'(\mathbb{R})$ of the same scaling degree, which is unique whenever $m^2 \neq (j+1)(j+2)$, $j = 0, \ldots, \lfloor \mathrm{sd}(u) + 1 \rfloor$.

2.2. Distributional solutions

Remark: Even though unique on-shell extensions as distributions generally exist, one cannot restrict these distributions to $z=0$. Instead one encode the information of the $c_1(\underline{k})$ and $c_2(\underline{k})$ modes in

$$\widehat{\Phi}_{\underline{k}}(z) = c_1(\underline{k})\sqrt{z}\, J_\nu\left(z\sqrt{-\underline{k}\cdot\underline{k}}\right) + c_2(\underline{k})\sqrt{z}\, Y_\nu\left(z\sqrt{-\underline{k}\cdot\underline{k}}\right)\,, \quad z>0\,.$$

though a suitable rescaling:

- $\Gamma_Y\left[\widehat{\Phi}_{\underline{k}}(z)\right] \doteq \lim_{z\to 0}\left[2^{-\nu}\,\Gamma(-\nu+1)\, z^{\nu-\frac{1}{2}}\,\widehat{\Phi}_{\underline{k}}(z)\right] = c_2(\underline{k})$;
- $\Gamma_J\left[\widehat{\Phi}_{\underline{k}}(z)\right] \doteq \lim_{z\to 0}\left[2^{\nu}\,\Gamma(\nu+1)\, z^{-\nu-\frac{1}{2}}\,\widehat{\Phi}_{\underline{k}}(z)\right] = c_1(\underline{k})$, if $c_2(\underline{k})=0$.

This can be applied at the level of the whole field and obtain at $z=0$ a *generalised free field* [Duetsch, Rehren (2003)].

2.2. Distributional solutions

Example. Scalar field in PAdS_4 with Dirichlet boundary conditions:

$$\Phi(z,x) = z^{3/2} \int \mathrm{d}^3\underline{k}\, J_\nu\left(z\sqrt{-\underline{k}\cdot\underline{k}}\right)\left[c_1(\underline{k})\, e^{-i\underline{k}\cdot\underline{x}} + c_1^\dagger(\underline{k})\, e^{i\underline{k}\cdot\underline{x}}\right] .$$

Applying the rescaling,

$$\begin{aligned}\hat{\Gamma}_J\left[\Phi(z,x)\right] &= \lim_{z\to 0}\left[2^\nu\,\Gamma(\nu+1)\, z^{-3/2-\nu}\Phi(z,x)\right]\\ &= \int \mathrm{d}^3\underline{k}\,\left(\sqrt{-\underline{k}\cdot\underline{k}}\right)^\nu\left[c_1(\underline{k})\, e^{-i\underline{k}\cdot\underline{x}} + c_1^\dagger(\underline{k})\, e^{i\underline{k}\cdot\underline{x}}\right]\\ &= \varphi^\partial(x)\,,\end{aligned}$$

is a generalised free field, the *boundary limit* of the scalar field. The field can be written as

$$\Phi(z,x) = z^{3/2+\nu} j_\nu\left(z^2\Box_{\mathbb{M}}\right)\varphi^\partial(x)\,,$$

where

$$j_\nu(u^2) = u^{-\nu} J_\nu(u)$$

is a polynomially bounded, convergent power series in u^2.

This provides an explicit expression for the scalar field Φ in AdS in terms of the generalised free field φ^∂ on the boundary.

2.2. Distributional solutions

Step 3. By means of a method of images, we extend the solutions to the whole Minkowski spacetime, exploiting the $z \to -z$ symmetry of the spacetime and of the field equation.

The procedure follows exactly as in [Dappiaggi, Nosari, Pinamonti (2014)].

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Next steps:

- construct the algebra of observables for the massive scalar field;
- construct Green operators and study their extensions to the boundary as bi-distributions;
- construct Hadamard states.

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1 Massless, conformally coupled scalar field on AdS

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3. Conclusions

- We constructed quantum states for a massless, conformally coupled scalar field in the Poincaré domain of AdS from quantum states of the Casimir-Polder system.
- We constructed distributional solutions for a massive scalar field in the Poincaré domain of AdS and studied the existence of on-shell extensions to the boundary as distributions. This system is equivalent to a scalar field system in the upper half-plane with a singular potential.
- Next steps:
  - construct observables and understand the notion of Green operators for the massive scalar field and if they can be extended to the boundary;
  - extend the procedure to asymptotically AdS spacetimes.

THANK YOU FOR YOUR ATTENTION!