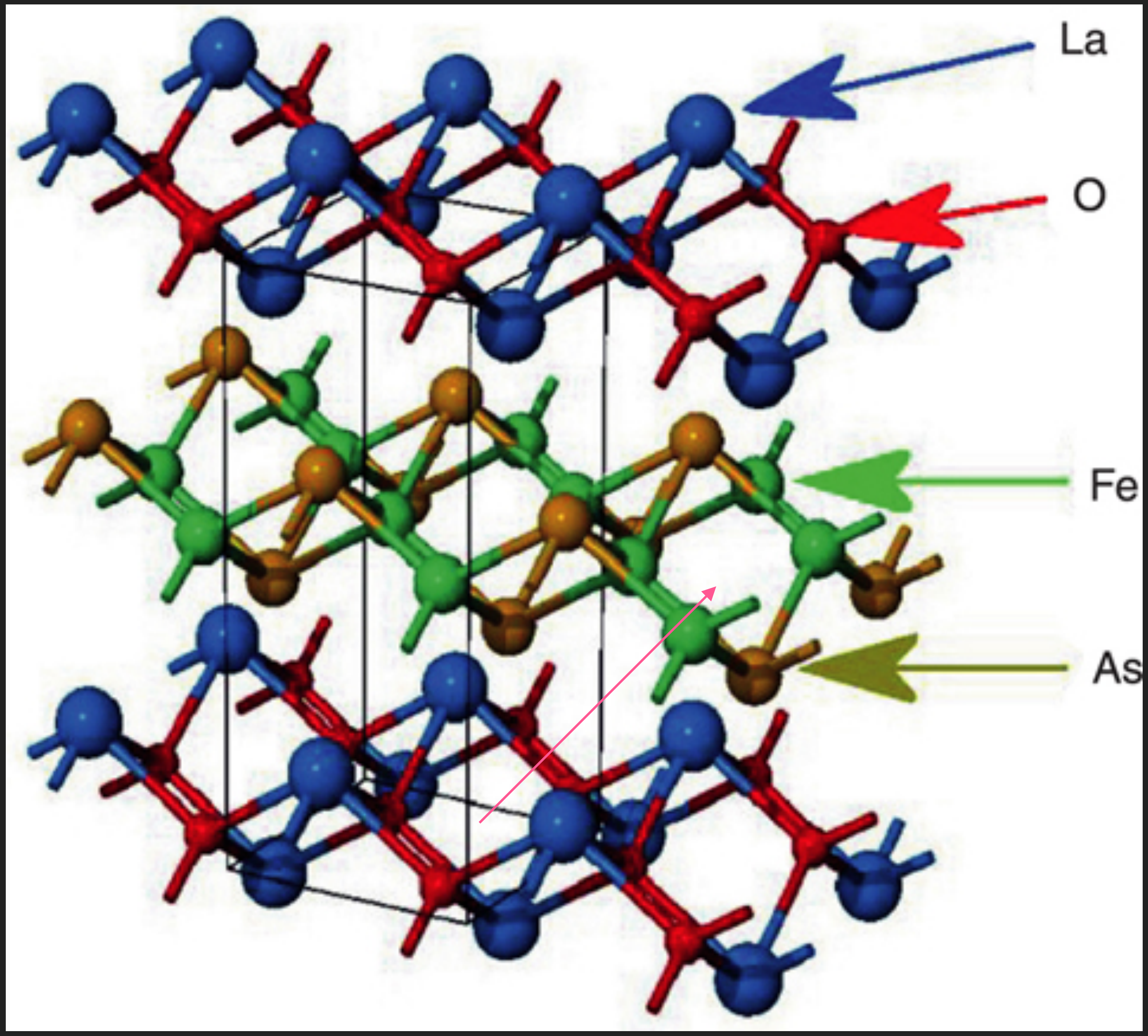
ROBERT HELLING (LMU)

HOLOGRAPHIC THEORIES: AN INVITATION

SUPER CONDUCTOR

MORE REALISTICALLY...



Cooper pair binding energy (kT_c): 1meV

Atomic binding energy (Coulomb): 100eV

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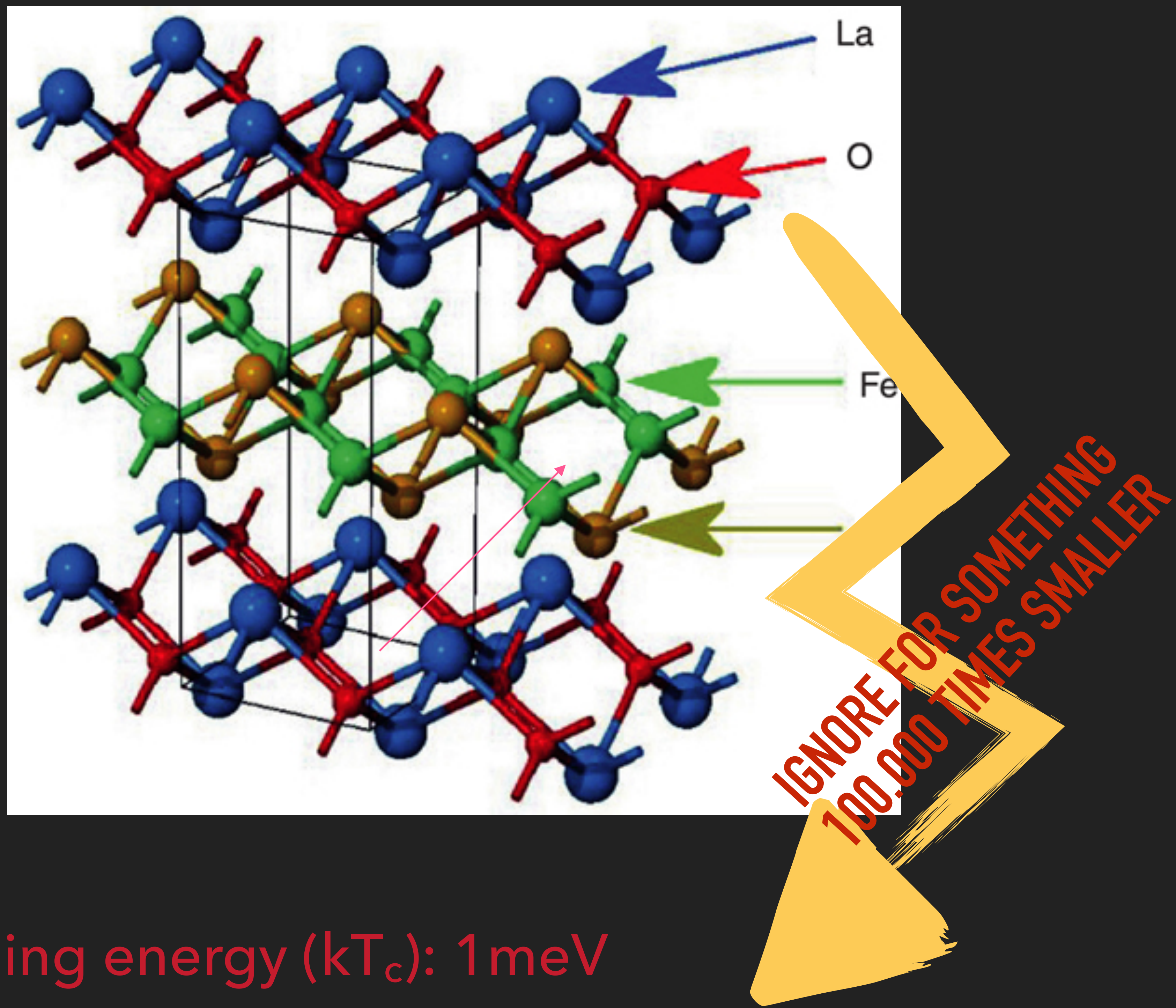

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Success Story

BCS IS AN EFFECTIVE THEORY

- Microscopic (UV) theory is know:

$$H = -\sum_i \Delta_i - \sum_{i,a} \frac{Z_a}{|R_a - r_i|} + \sum_{i<j} \frac{1}{|r_i - r_j|}$$

- Superconductivity is a phenomenon of the IR end of the renormalization group flow
- There is little control over the RG flow, so BCS as an "effective theory"

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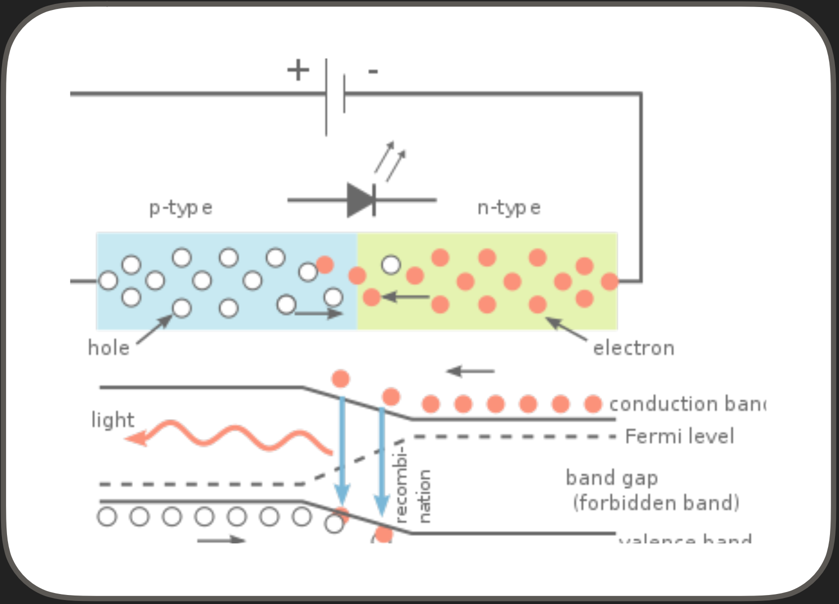

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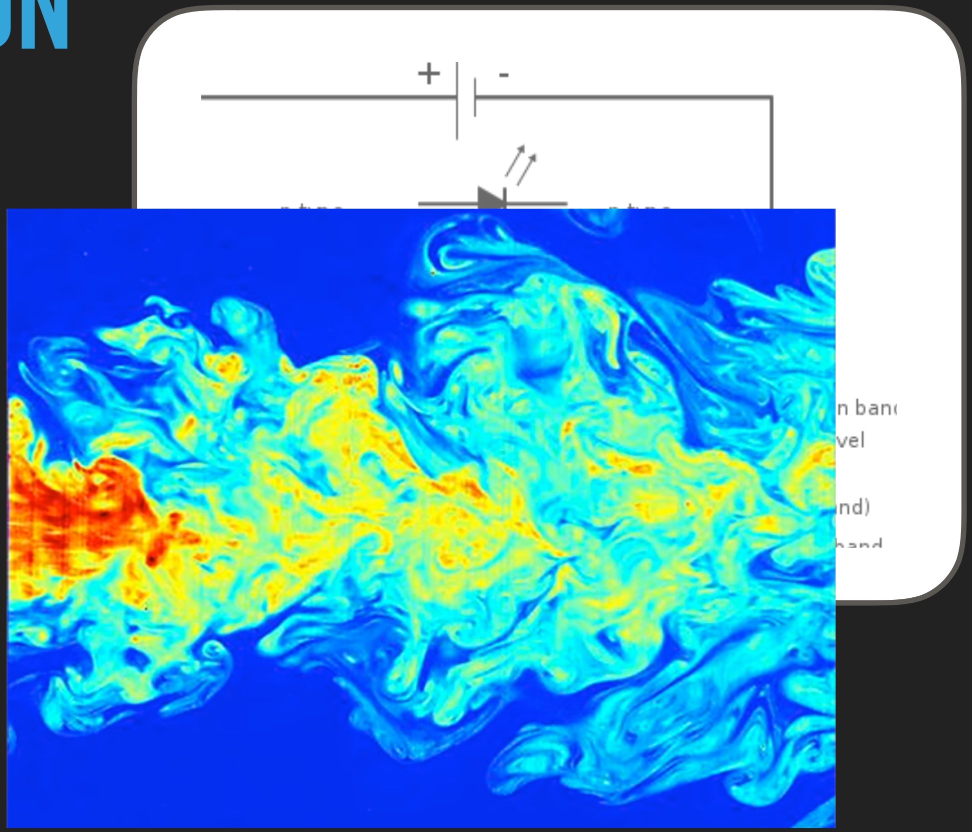

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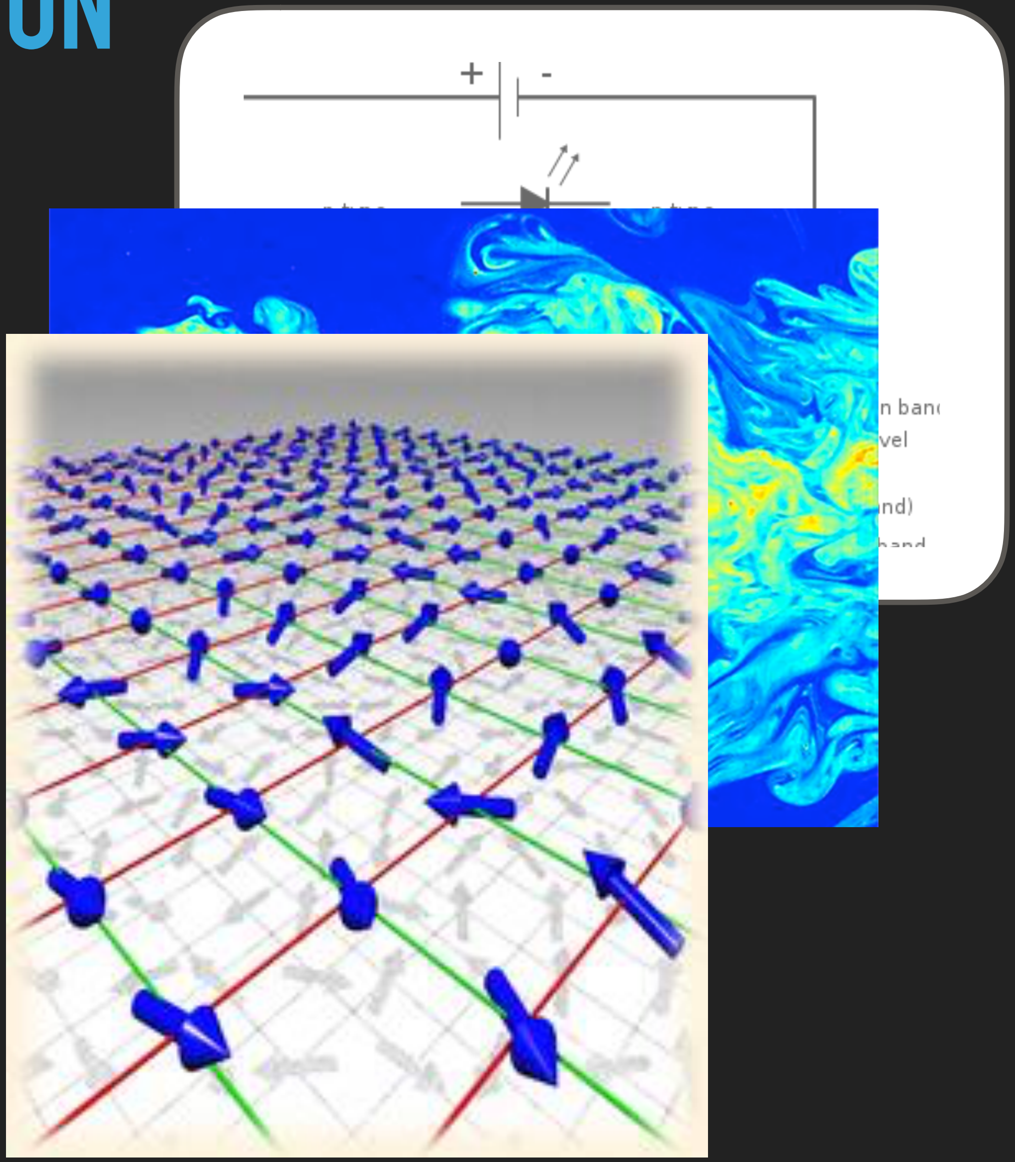
- Ferromagnets



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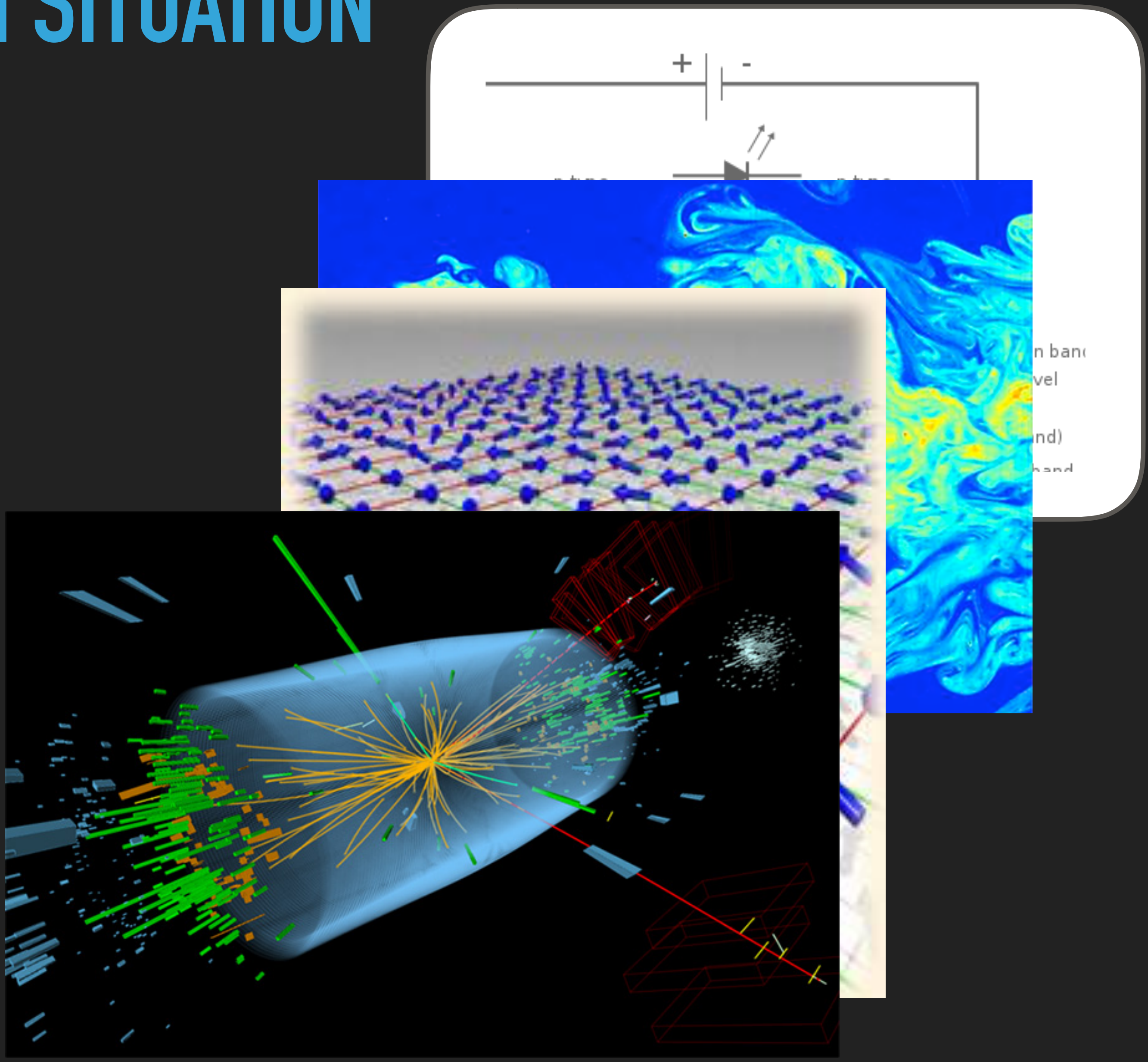
- Fermi Liquid
- Hydrodynamics
- Ferromagnets
- Standard Model



WHAT THE PHYSICIST NEEDS…

- What is thought for is a "simpler" theory that makes qualitative and quantitative predictions for low energy phenomena (e.g. transport properties like conductivity)

- In the end, in comes down to guessing a quantum (field) theory (a model).

- To have a starting point, we need a language to write down the QFT.

- Output are n-point (Wightman) functions, e.g.

$$\sigma_{\alpha\beta}(\omega) = \frac{ine^2}{m\omega}\delta_{\alpha\beta} + \frac{1}{\hbar\omega V}\int_0^\infty dt\, e^{i\omega t}\langle[j_\alpha(t), j)_\beta(t)]\rangle$$

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- Holography

ANTI-DE-SITTER-SPACE

$$ds^2 = \frac{L^2}{r^2}\left(dr^2 + \eta_{\mu\nu} dx^\mu dx^n\right)$$

Classical bulk action

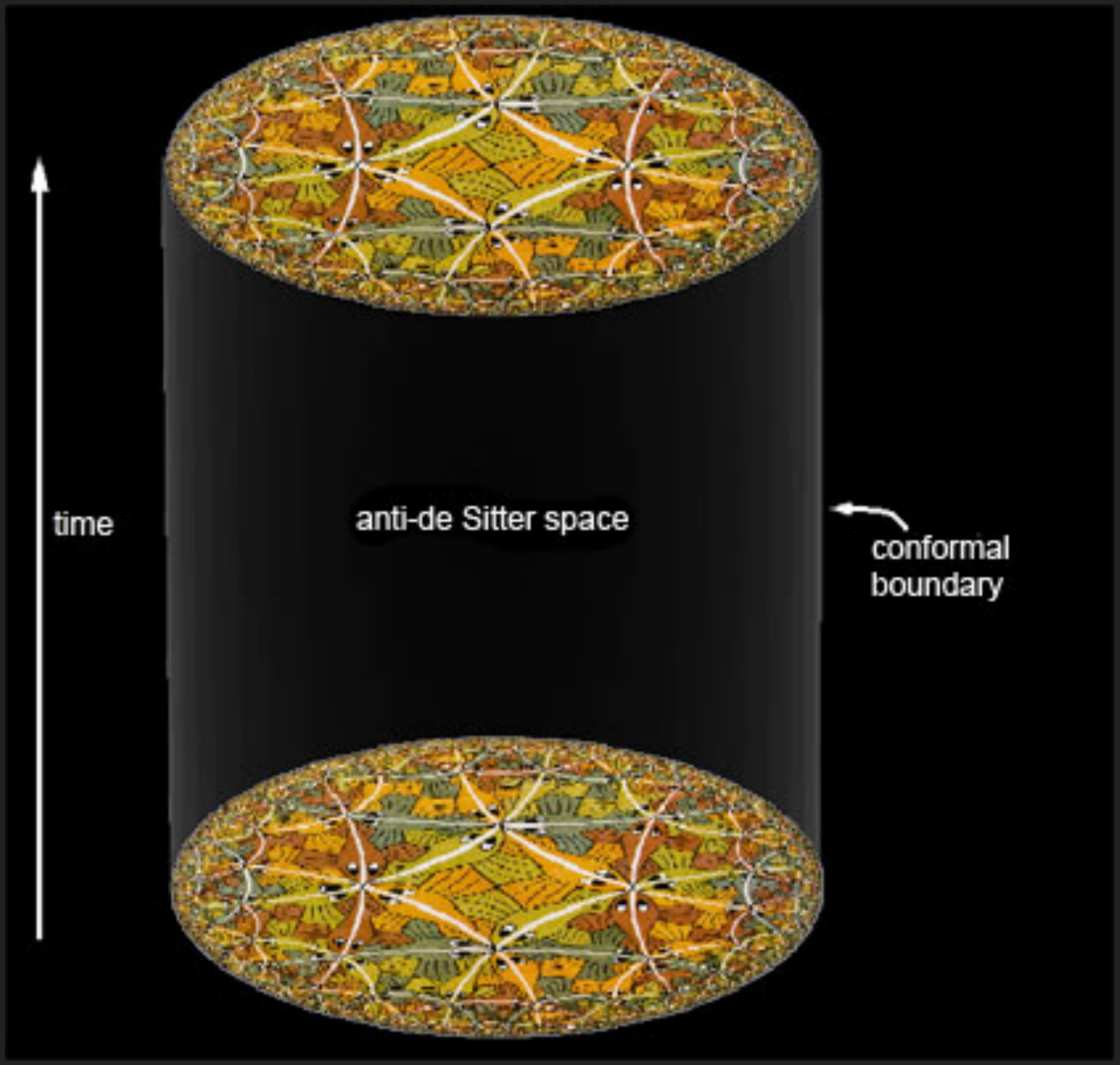
with asymptotic prescription

computes

n-point functions of

QFT on the boundary

$$\left\langle e^{\mathcal{O}(\phi_0)} \right\rangle_{QFT} = e^{i\mathcal{S}_{bulk}}\Big|_{``\phi\to\phi_0"}$$



ADS/CFT DICTIONARY

- Pick a classical bulk theory (gravity + other fields (scalars, vectors, etc.). Require solutions with asymptotic ($r \to 0$) AdS geometry (e.g. AdS-Black hole for T>0)
- For each field, there is a corresponding operator in the boundary theory, e.g.

$$A_\mu(x,r) \leftrightarrow J_m(x)$$
$$g_{\mu\nu}(x,r) \leftrightarrow T_{mn}(x)$$

AN EXAMPLE: A SCALAR FIELD

- For $\phi(x,r) = \phi(r)e^{ikx}$ Klein-Gordon equation becomes

$$-r^{d+1}\partial_r\left(r^{-d+1}\partial_r\phi\right) + \left(k^2r^2 + m^2L^2\right)\phi = 0$$

- Near the r=0 boundary,

$$\phi(r) = \left(\frac{r}{L}\right)^{\Delta_-}\left(\phi_0(x) + \cdots\right) + \left(\frac{r}{L}\right)^{\Delta_+}\left(\phi_1(x) + \cdots\right)$$

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2L^2}$$

RECIPE

$$\phi(r) = \left(\frac{r}{L}\right)^{\Delta_-} (\phi_0(x) + \cdots) + \left(\frac{r}{L}\right)^{\Delta_+} (\phi_1(x) + \cdots)$$

- Fix $\phi_0(x)$ using the boundary source (test function)
- Impose a boundary condition at large r (typically: purely in-going at BH horizon)
- Solve the bulk equations of motion
- Read off $\phi_1(x)$ as linear response

EXAMPLE: ELECTRIC CONDUCTIVITY

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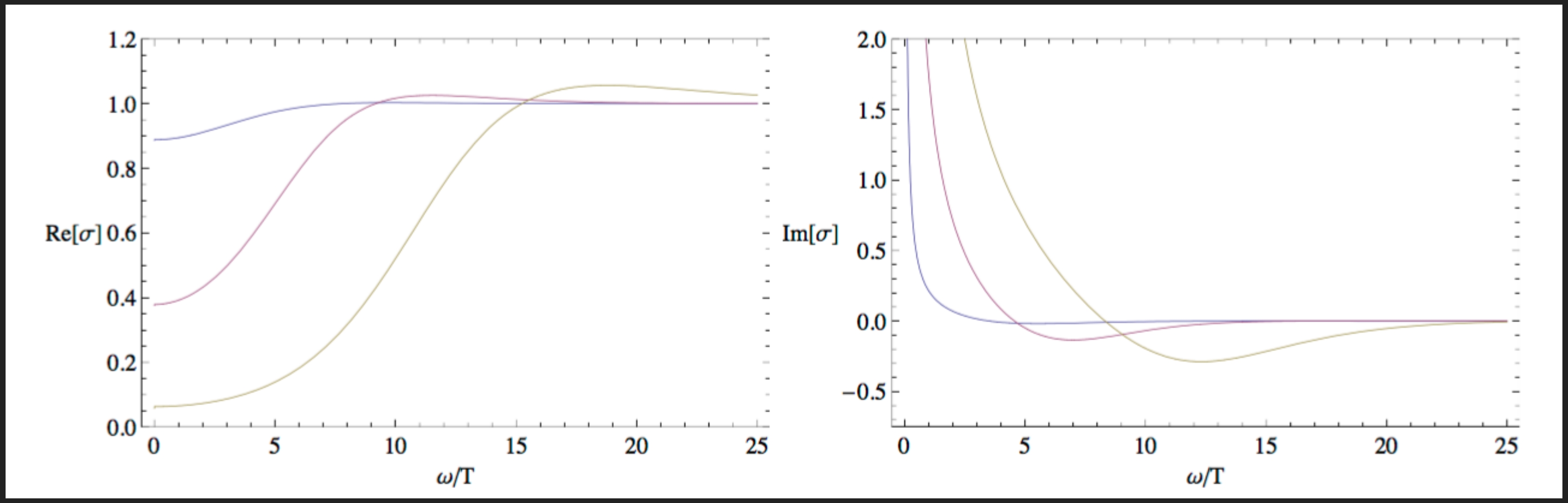

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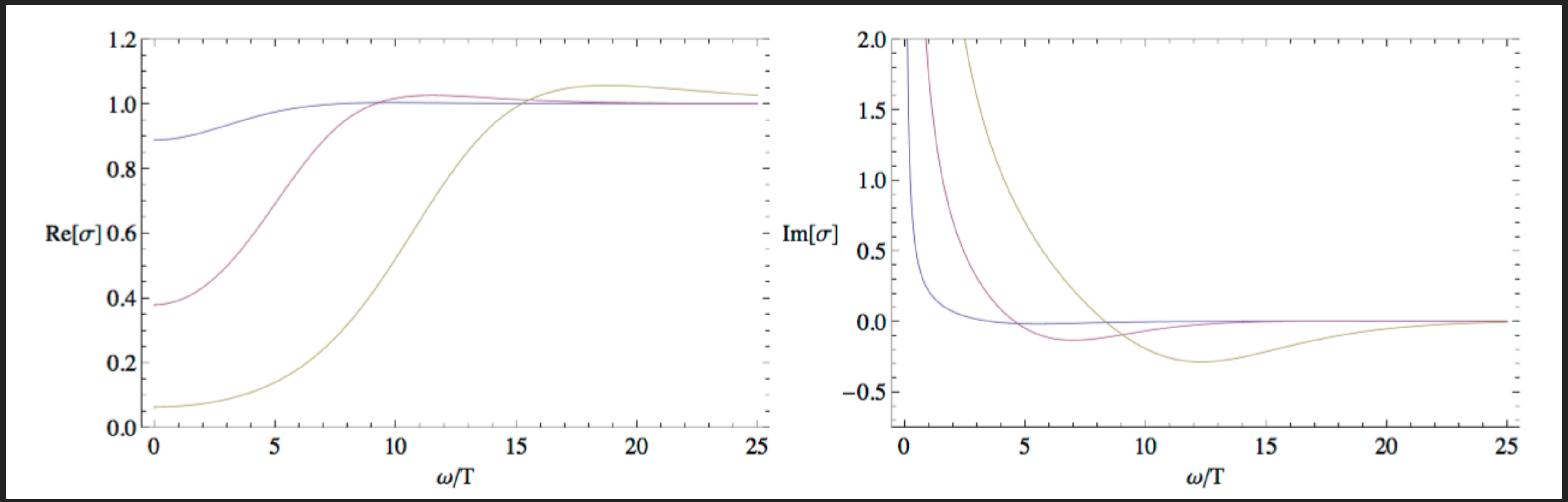
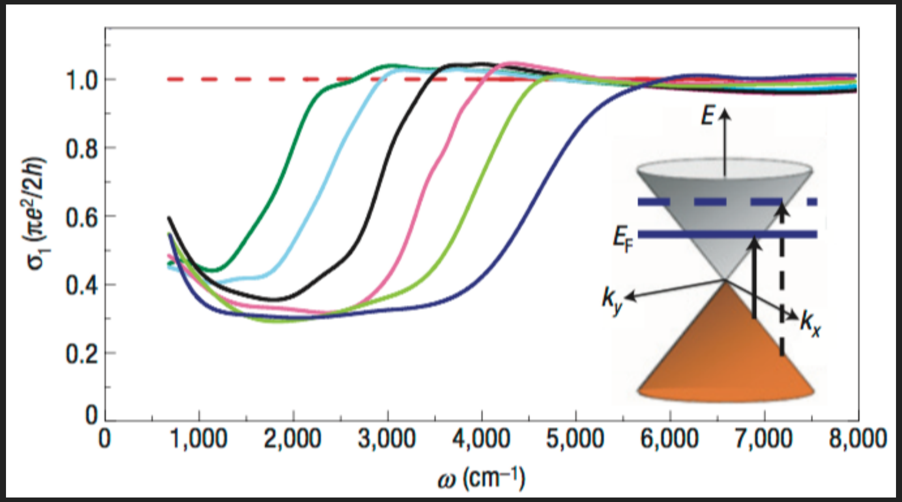
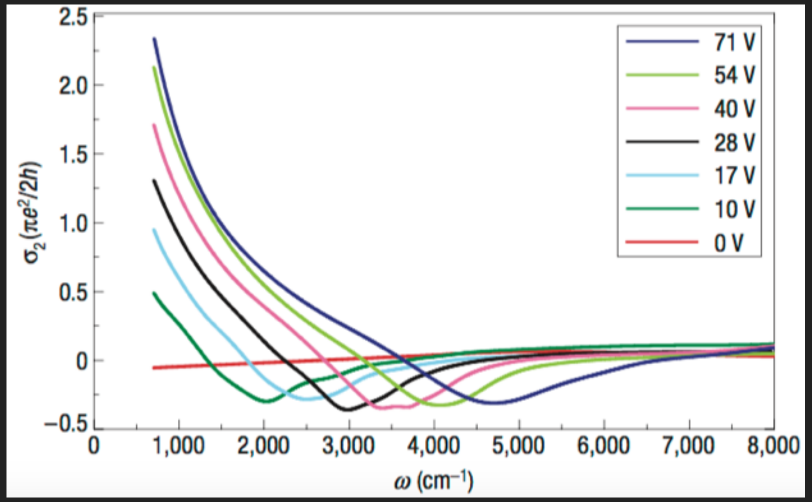


- Compare to graphene (measured):





HOLOGRAPHIC REALIZATIONS OF EFFECTIVE FIELD THEORIES

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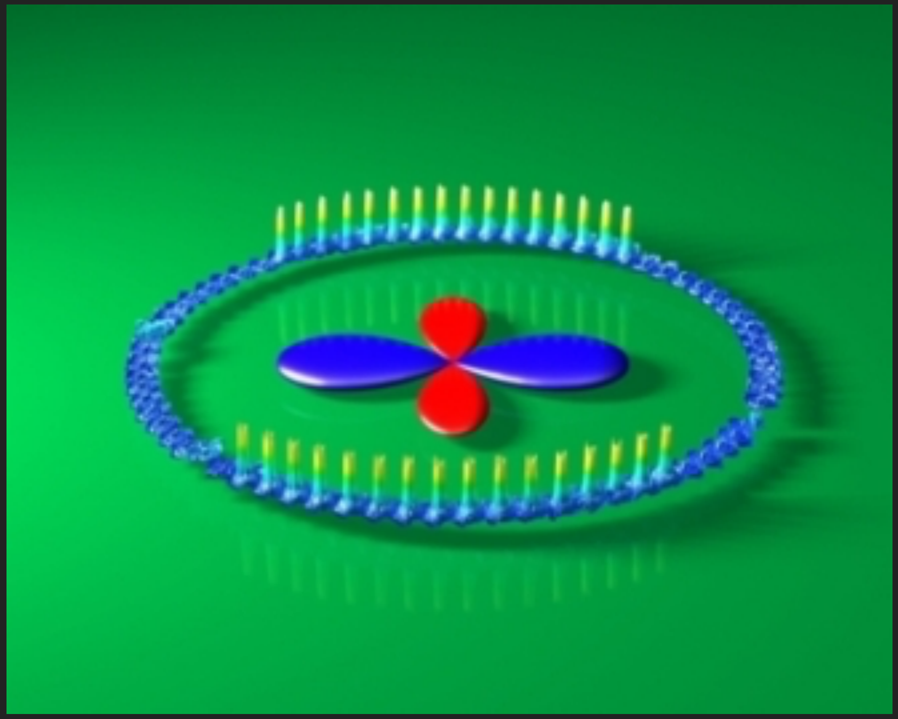

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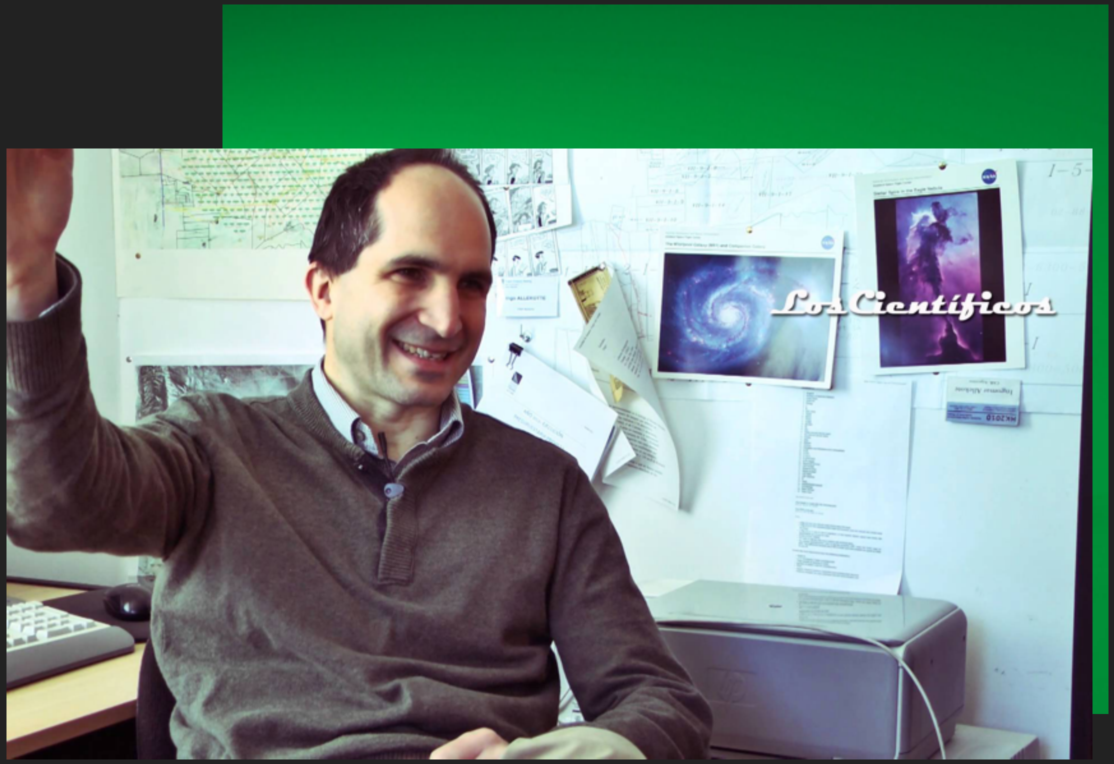

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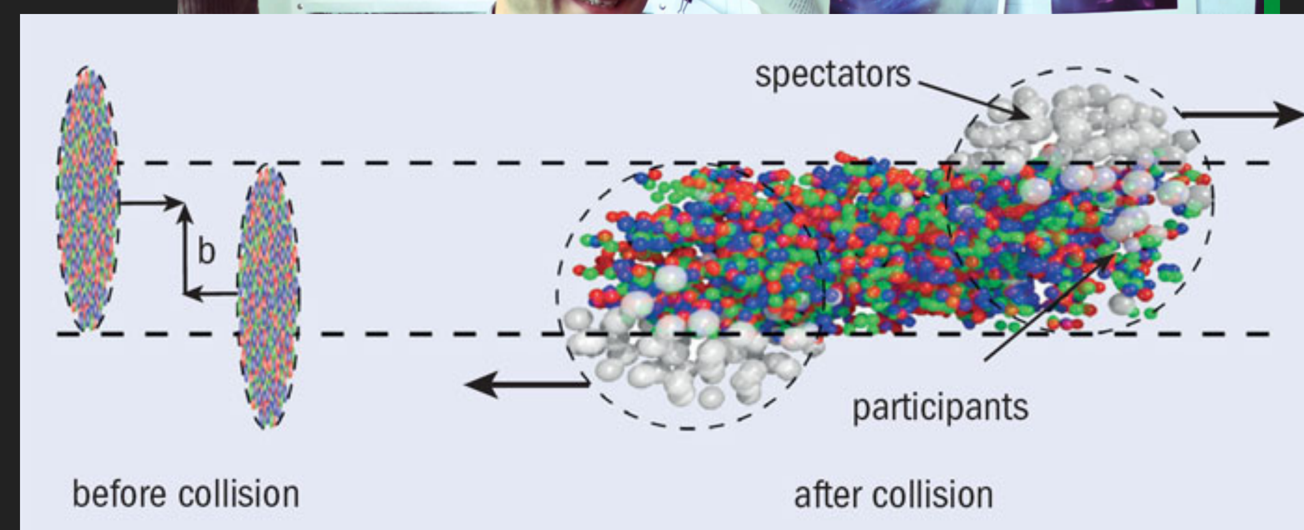

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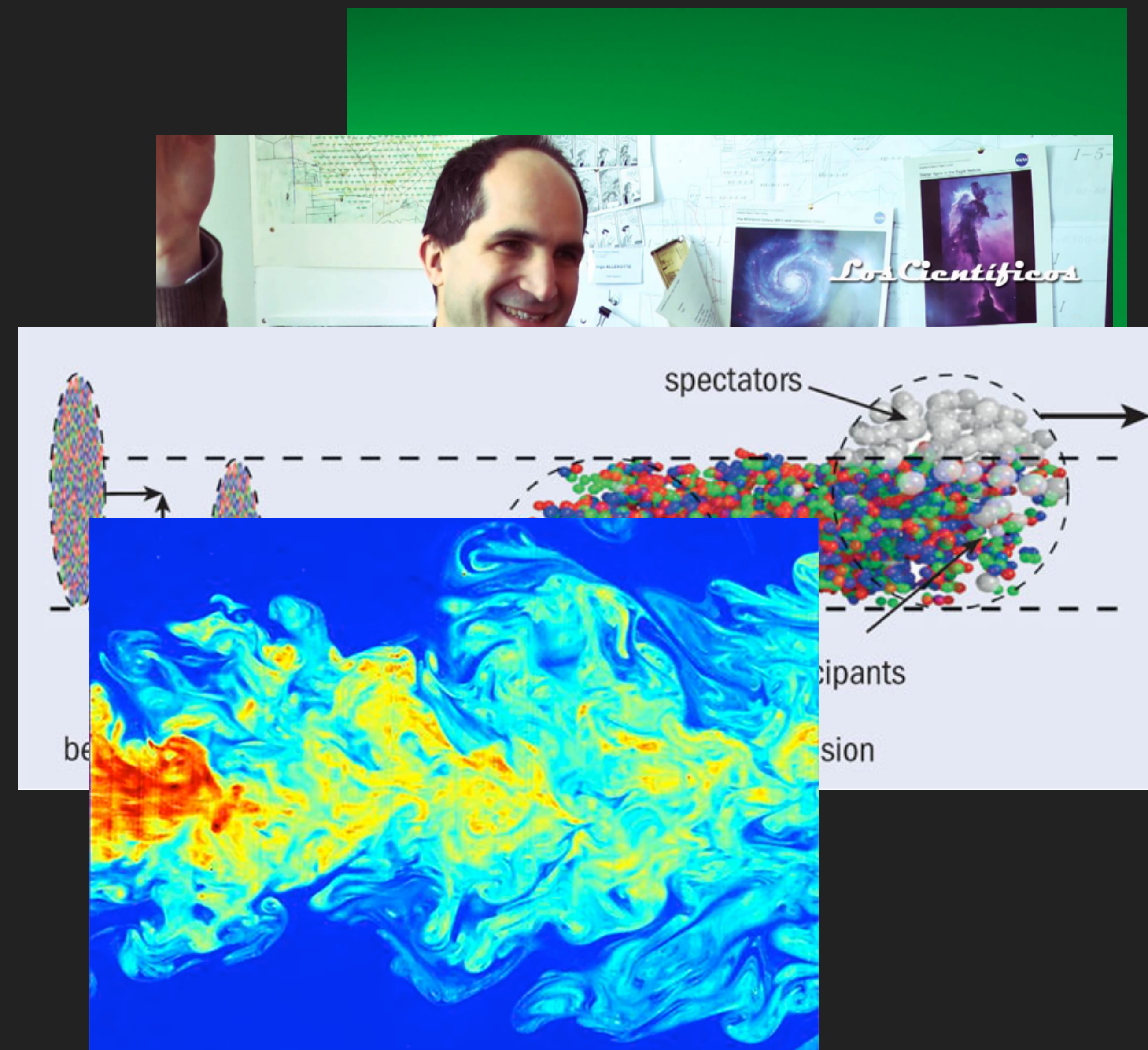

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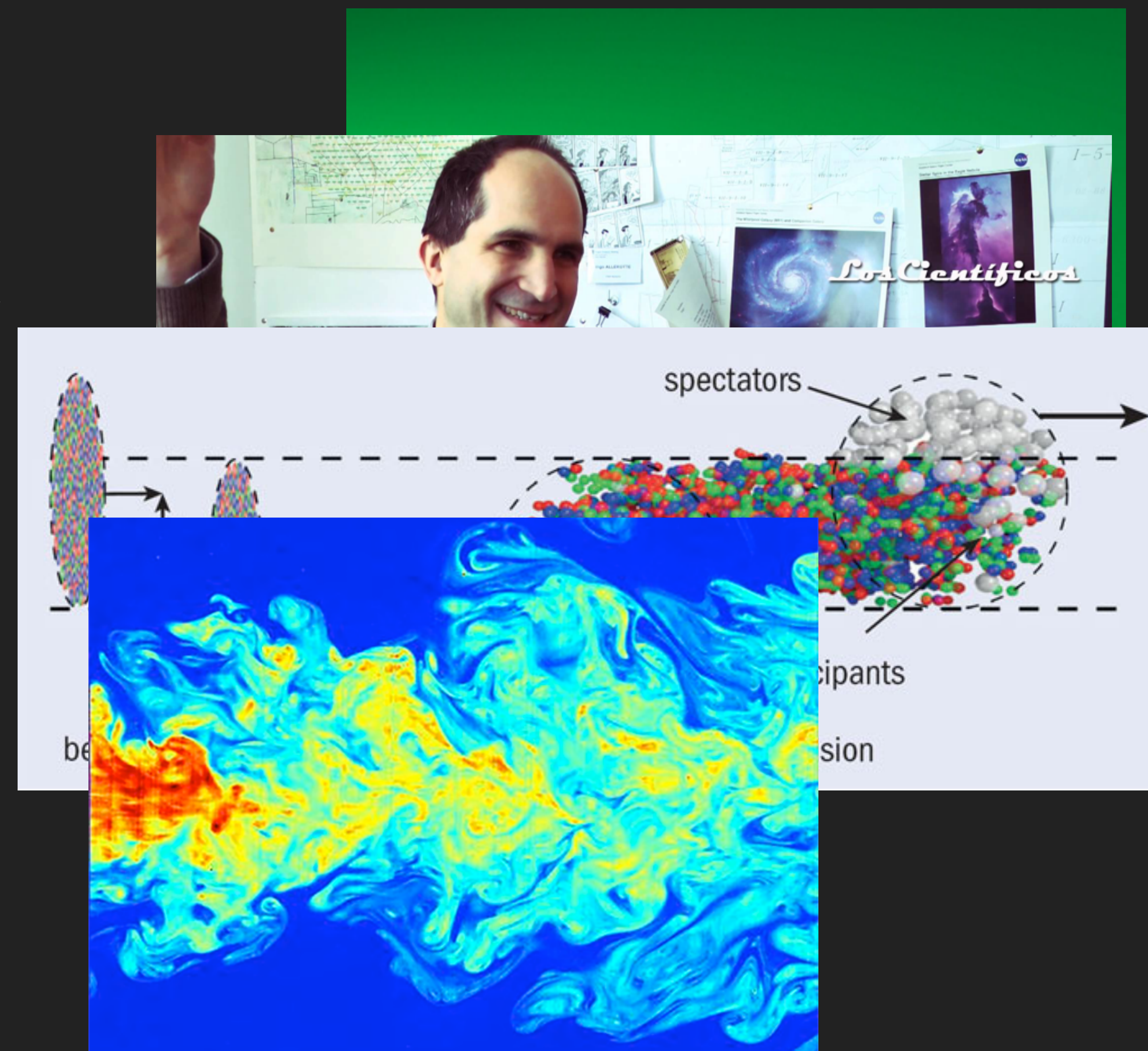

MY QUESTION TO YOU

- Does this approach to correlation functions really encode a QFT (à la Wightman, Osterwalder-Schrader etc)?
- If yes, alternative route to free fields and 2d CFT
- If no, interesting to the holographic community

FURTHER READING

- Sean A. Harntoll „Lectures on holographic methods for condensed matter physics" arxiv:0903.3264 (source for conductivity plots)
- David Tong "Lectures on Holographic Conductivity" http://www.damtp.cam.ac.uk/user/tong/talks/zakopane.pdf
- Kostas Skenderis, Balt C. van Rees "Real-time gauge-gravity duality: Prescription, Renormalization and Examples" arXiv:0812.2909