Evidence for a modified LSZ condition and asymptotic field algebra in QED

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"A direct approach to the critical points of the infrared problem in QED"

Status of the infrared problem...

A. How may asymptotic charged fields be constructed?

B. What can be said on - their dynamics
- their algebraic relations with the asymptotic electromagnetic field?

Recall: Construction af massless asymptotic fields done by Buchholz (in the vacuum sector)

But charged fields cannot be constructed a la LSZ (Haag-Ruelle)

Absence of a gap covered by Dybalski, but

Basic problem: the absence of 1 particle shell $p^2 = m^2$,

a consequence of Gauss' law [Buchholz]

A recent advance on asymptotic charged states in models of QED:

Chen, Frohlich, Pizzo: Non-relativistic QED for one charged particle

- analysis based on the construction of pure particle states

- main difficulty: energy-momentum relation for charged particles

(bottom of energy spectrum, fundamental because photon emission depends on the asymptotic velocity).

However: for a relativistic theory, the relativistic relation holds automatically [Borchers-Buchholz]

- modified LSZ formulas are given after and in terms of that construction (for one charged particle)

We explore the possibility of a direct Scattering Theory approach.

Failure of LSZ construction already for Coulomb scattering

Solved by Dollard asymptotic dynamics

Do Dollard's ideas really apply in field theory models?

Kulish-Faddeev-Rohrlich approach

Dollard (Zwanziger) define asymptotic fields?

Dollard asymptotic dynamics is not a group!

In conclusion, the basic questions about the Dollard approach are:

- Is Dollard scattering theory compatible with space-time covariant asymptotic fields ?

- Can Kulish-Faddeev-Rohrlich be confronted with a Heisenberg formulation of scattering, asymptotic fields and asymptotic condition?

Program: obtain information from:

1. A general analysis of "Dollard Scattering Theories"

2. Their application in models of QED

1. Dollard Scattering Theories:

The problem: exhibit the implications on asymptotic fields and dynamics of Dollard approach:

$$U(t)\psi \sim U_D(t)\varphi \quad \text{for} \quad t \to \pm\infty \, ,$$

with $U_D(t)$ a (strongly continuous) family of unitary operators (not a group), ψ in a "scattering subspace" of $\mathcal{H}$, φ in $\mathcal{H}$ (or in a "reference scattering space"), i.e.

$$U(-t)U_D(t) \quad \text{converges strongly} \, , \quad t \to \pm\infty$$

defining isometries $\Omega_\pm$.

The results:

- Existence of Moller operators implies existence of (strongly continuous) groups, $U_\pm(s)$, asymptotic on the right to the Dollard dynamics,

$$U_D(t+s) \sim U_D(t)\, U_\pm(s) \quad \text{for} \quad t \to \pm\infty$$

$U_\pm(s)$ are unique and given by

$$U_\pm(s) = \lim_{t\to\pm\infty} U_D(t)^{-1}\, U_D(t+s)$$

- The Møller operators interpolate between U and $U_\pm$

$$U(s)\,\Omega_\pm = \Omega_\pm\, U_\pm(s)$$

- Even if the Dollard Hamiltonian

$$i\, d/dt\, U_D(t) = H_D(t)\, U_D(t)\,,$$

is "asymptotic" to a free Hamiltonian for $t \to \pm\infty$, $U_\pm$ needs not to be the free. In this case the free dynamics is asymptotic "on the left", a notion not implying uniqueness. (However, this implies $\sigma(H_\pm) \subset \sigma(H_0)$)

- Heisenberg asymptotic variables are defined by

$$A_{out/in} \equiv \Omega_\pm\, A\, \Omega_\pm^*$$

on the scattering subspace

- By the interpolation formulas

$$H = H_{\pm}(asymptotic \;\; variables)$$

- Dollard corrections to LSZ formulas:

$$A_{out/in} \equiv \lim_{t \to \pm\infty} U(t)^* \left(U_D(t) A U_D^*(t) \right) U(t)$$

- The same results can be reproduced for an adiabatic formulation of scattering, with some care on "mass renormalization terms"

Conclusions on Dollard Scattering Theories:

- Distinction between "Dollard dynamics" and "asymptotic dynamics". The group property is lost only at a "subleading" level

- Recovery of the "usual" asymptotic variables and dynamics

- Reinterpretation of Dollard dynamics as corrections to LSZ procedures

- Possibly, with consequences on the dynamics of asymptotic fields

Notice: a non-free asymptotic dynamics is essential for an application to models with vacuum and without (charged particle) states of definite mass

2. Dollard's approach to QED:

The problem: Control the application of the above Dollard strategy in relevant field theory models of QED.

Our model, with "heavy classical charged particles"

- is covariant under space-time translations

- has a non-trivial, non-soluble, dynamics of charged particles

- has a realistic (velocity dependent) photon interaction, reproducing the asymptotic coherent states of FCP

- includes Coulomb interactions

- allows for a "relativistic" particle dynamics, avoiding the necessity of constraints on velocities

The difficulties of the energy-momentum dispersion problem are avoided by neglecting photon recoil

Photon recoil whould give corrections to trajectories of the same (logarithmic) asymptotic form as the Coulomb effects.

The model:

In $L^2(\Gamma) \times \mathcal{F}$, Γ the phase space, $\mathcal{F}$ the Fock space of free photons

Classical eq. of motion for q_i, p_i, with Coulomb interactions; the electromagnetic field interacts with their current.

Koopman formulation of the dynamics, with

$$H = h_0 + h_I + H_0 + H_I + \Delta E$$

$$h_0 = \sum_i v_i \, P_i \ , \quad h_I(q, Q) = \frac{\partial V}{\partial q_i} Q_i$$

$$P_i \equiv -i\,\frac{\partial}{\partial q_i}\ ,\quad Q_i \equiv i\,\frac{\partial}{\partial p_i}$$

$$v(p) = p/m \quad \text{or} \quad v(p) = \frac{p}{(p^2+m^2)^{1/2}}$$

$$V(q) = \sum_{j\neq i} \frac{e_i e_j}{8\pi(|q_i - q_j|^2 + a^2)^{1/2}}$$

$$H_I(a, a^*, q, p) = \frac{1}{(2\pi)^{3/2}} \sum_{i,\,\lambda} e_i$$

$$\int \frac{d^3k}{\sqrt{2|k|}}\,\eta(k)\,[a(k,\lambda)\,\epsilon(\lambda)\,v(p_i)\,e^{ikq_i} +\ h.c.]$$

$$\Delta E(p) = \sum_i e_i^2\,\delta E(p_i)$$

Adiabatic formulation: $e_i \to e_i\, e^{-\varepsilon|t|}$

Dollard Hamiltonians

$$H = h_0 + h_I^D(t) + H_0 + H_I^D(t) + \Delta E(p)$$

$$h_I^D = h_I^{a=0}(q = vt, Q = \dot{Q}t)$$

$$H_I^D = H_I(a, a^*, v(p)t, p)$$

Explicit Dollard dynamics $\;U^D(t) = u^D(t)\,\mathcal{U}^D(t)$

Results:

Dynamics $\quad U(t) = u(t)\,\mathcal{U}(t)(\{q_s\})$

Particle Møller operators

$$\omega_\pm = s - \lim u(t)^* u^D(t)$$

Adiabatic Møller operators

$$\Omega_\pm = s - \lim_{\varepsilon\to 0}\lim_{t\to\pm\infty} U(t)^{\varepsilon\,*}\,U^{D\,\varepsilon}(t) = W_\pm\,\omega_\pm$$

$$W_\pm = \lim_{\varepsilon\to 0} W^\varepsilon_{0\pm}(\{q_s\}(q,p))\,W^{\varepsilon\,*}_{D\pm}(p_\pm(q,p))$$

well defined on $\omega_\pm L^2(\Gamma)\times\mathcal{F}\equiv\mathcal{H}_\pm$

$W^\varepsilon_{D\pm}(p))$ implementing shifts which converge to

$$\alpha_{as}(a^*(k,\lambda)) = a^*(k,\lambda) + \sum_i J^D(k,\lambda,p_i)$$

$$J^D(k,\lambda,p) = \frac{e}{(2\pi)^{3/2}}\,\frac{\epsilon_\lambda(k)\,\eta(k)\,v(p)}{(2|k|)^{1/2}(|k| - v(p)k)}$$

Asymptotic dynamics

$$U_\pm(s) \equiv \lim_{\varepsilon\to 0}\lim_{t\to\pm\infty} U^{\varepsilon\,*}_D(t)\,U^\varepsilon_D(t+s)$$

$$U_+(s) = U_-(s) = u_0(s)\,\alpha_{as}(\mathcal{U}_0(s))$$

Interpolation formulas

$$H\,\Omega_\pm = \Omega_\pm\,H_{as}\ ,\quad H_{as} = h_0 + \alpha_{as}(H_0)$$

The same for the generators of space translations, with

$$P_{as} = -i \sum_n \partial/\partial q_n + \alpha_{as}(P_{ph})$$

From Møller operators and interpolation formula, on $\mathcal{H}_{\pm}$:

Heisenberg asymptotic canonical variables

$$a_{out/in}(k) \equiv \Omega_{\pm}\, a(k)\, \Omega_{\pm}^{*} \ , \quad q_{out/in} \ , \quad p_{out/in}$$

Expression for the Hamiltonian in terms of asymptotic variables

$$H = \Omega_{\pm}\, (h_0 + \alpha_{as}(H_0))\, \Omega_{\pm}^{*}$$

$$= h_0(q_{out/in}, p_{out/in}) + (\alpha_{as} H_0)(a^{\#}_{out/in}, p_{out/in})$$

The same for the generators of space translations

Field theory reformulation, LSZ asymptotic formulas

Two main points:

- Convergence of the Dollard Møller operators implies a convergent modified (adiabatic) LSZ formula for charged fields

- The asymptotic limit of the electromagnetic fields does not need Dollard corrections

(Freedom: "The LSZ (Haag-Ruelle) constructions have nothing to do with a reference unitary evolution")

Convergence of the ordinary (massless LSZ) construction for the electromagnetic fields is easy in the model:

$$W(f, \lambda) \equiv e^{-i(a(f,\lambda)+h.c.)} \;, \quad f_t(k) \equiv f(k) e^{-i\,|k|\,t}$$

$$W(f_{-t}, \varepsilon, t) \equiv U^{\varepsilon\,*}(t)\, W(f_{-t})\, U^{\varepsilon}(t)$$

Introducing the Dollard dynamics and using, on $\mathcal{H}_{\pm}$, for $F_t^{\varepsilon}(p) \;\rightarrow\; F_{\pm\infty}(p)$ uniformly for bounded p,

$$e^{i\rho_t^{\varepsilon}(F_t^{\varepsilon})} \longrightarrow e^{i\rho_{out/in}(F_{\pm\infty})}$$

it follows

$$W(f_{-t}, \varepsilon, t) \rightarrow \Omega_{\pm}\, W(f, \lambda)\, \Omega_{\pm}^{*}\, e^{i\rho_{out/in}(J^D(f,\lambda)+h.c.)}$$

defining

$$b^*_{out/in}(f,\lambda) = a^*_{out/in}(f,\lambda) + \rho_{out/in}(J^D)(f,\lambda)$$
$$= \alpha_{as}(p_{out/in})\,(a^*_{out/in}(k,\lambda))$$

Implications on the Hamiltonian and on the generators of space translations

$$H = h_0(q_{out/in}, p_{out/in}) + (\alpha_{as} H_0)(a^{\#}_{out/in}, p_{out/in})$$
$$= h_0(q_{out/in}, p_{out/in}) + H_0(b^{\#}_{out/in})$$
$$P = -i \sum_n \partial/\partial q_{out/in\, n} + P_{ph}(b^{\#}_{out/in})$$

The photon fields $b^{\#}_{out/in}$ commute with h_0 and are free (canonical) fields

Charged fields (fermion case) are defined on

$$\sum_n L^2_{ant}(\mathrm{IR}^{6n}) \oplus \mathcal{F}$$

$$(\Phi(f)^*\psi^n)(q, q_1 \ldots q_n, p, p_1 \ldots p_n) =$$
$$= \sqrt{(n+1)}\,(f(q,p)\psi^n(q_1 \ldots p_n))_{ant}$$

with partial Fourier transform

$$\Psi^*(P,p) = (2\pi)^{-3/2} \int d\,q\, e^{iqP}\Phi^*(q,p)$$

They satisfy

$$\{\Psi(P,p),\ \Psi^*(P',p')\} = \delta(P-P')\,\delta(p-p')$$
$$\{\Psi(P,p),\ \Psi(P',p')\} = 0$$

Heisenberg asymptotic charged fields:

$$\Psi^*_{out/in}(f) \equiv \Omega_\pm\,\Psi^*(f)\,\Omega^*_\pm$$

on $= \Omega_\pm\,\mathcal{H}$

Asymptotic algebra: From

$$[\Psi^*_{out/in}(P,p)\,,\ a^*_{out/in}(k,\lambda)] = 0$$
$$[\rho_{out/in}(p)\,,\ a^*_{out/in}(k,\lambda)] = 0$$
$$[\rho_{out/in}(p')\,,\ \Psi^*_{out/in}(P,p)] = \delta(p'-p)\,\Psi^*_{out/in}(P,p)$$

it follows

$$[b^\#_{as}(k,\lambda), \Psi^*_{as}(P,p)] = J^D(k,\lambda,p)\,\Psi^*_{as}(P,p)$$

Even if

$$H = h_0(\Psi^\#_{out/in}) + H_0(b^\#_{out/in})$$

$\Psi^\#_{out/in}$ is not free, by its commutation relations with $b^\#_{out/in}$

Computation of the (Dollard) modified LSZ formula:

The Dollard dynamics of the charged fields is

$$\Psi_D^{\varepsilon\,*}(f,t) = \int dP\,dp\,f_{-t}(P,p)\,\Psi^*(P,p)$$

$$e^{-i[a(F_t^{D\varepsilon}(p),t)+h.c.]}\;e^{i\rho\,(\chi_t^\varepsilon(P,p))}$$

with

$$f_t(P,p) = e^{iPvt}\,f(P,p)$$
$$F_t^{D\varepsilon}(k,\lambda,p) \to -iJ^D(k,\lambda,p)$$
$$\chi_t^\varepsilon(P,p,P',p') = L_t^\varepsilon(p,p') + C_t(P,p,P',p')$$

The strong convergence of the (adiabatic) Møller operators then implies, on $\Omega_\pm\mathcal{H}$,

$$\Psi^*_{out/in}(f) = \lim \int dP\,dp\,f_{-t}(P,p)\,\Psi_t^{\varepsilon\,*}(P,p)$$

$$e^{i\rho_t^\varepsilon\,(\chi_t^\varepsilon(P,p))}\;e^{-i\int_0^t ds\,A_t^\varepsilon\,(\overleftrightarrow{\partial}_t G_{t-s}*j^\varepsilon(v(p);s))}$$

with

$$j_\mu^\varepsilon(v;x,s) \equiv e\,v_\mu\tilde{\eta}(x-vs)\,e^{-\varepsilon|s|}\;,\quad v_\mu \equiv (1,v)$$

All the corrections to the LSZ formula are given by the Dollard dynamics. The Weyl exponential is the x space expression for the above exponential of a, a^*.

Alternative version in terms of asymptotic electromagnetic fields

The above corrections can be written (i.e. the same limit is obtained) in terms of asymptotic (free) electromagnetic potentials $B_{out/in}$ and asymptotic particle density:

$$\Psi^*_{out/in}(f) = \lim \int dP\, dp\, f_{-t}(P,p)\, \Psi_t^{\varepsilon\,*}(P,p)$$

$$e^{i\rho_t^{\varepsilon}\,(C_t(P,p))}\; e^{i\rho_{out/in}\,(L_{\pm}^{\varepsilon}(p)+c(p))}\; e^{-iB_{out/in}(j_{\pm}^{\varepsilon}(v(p)))}$$

Heisenberg fields without adiabatic switching

Asymptotic limit of electromagnetic fields: a time smearing (Herbst, Buchholz)

$h_T(t) \equiv 1/c(T)\, h((t-T)/c(T))\ , \quad c(T) \to \infty$

gives the same (free) asymptotic fields $B_{out/in}$

For charged fields:

Non trivial role of Coulomb logarithmic contributions to $\{q_s(q,p)\}$ for photon emission, not canceled by the photon Dollard dynamics, constructed on straight lines

In presence of momentum exchange, similar logarithmic distortions of trajectories are also produced by photon emission

No solution from $h_T(t)$

Solution: modification to the LSZ formula, using position variables

Replace in the electromagnetic corrections to the LSZ formula,

$$p_t \to q_t/t$$

Equivalently, replace the string

$$(sv(p_t), s) \to (sv(q_t/t), s) \;, \quad s \in [0, t]$$

The same replacement, together with $P_t \to Q_t/t$, can be done in the Coulomb and Lienard-Wiechert terms, with no effect in the asymptotic limit

Resulting LSZ formulas:

$$\Psi^*_{out/in}(f) = \lim_t \int dQ\, dq\, \tilde{f}_{-t}(Q, q)\, \Psi^*_t(Q, q)$$

$$e^{i\rho_t\,(\chi_t(Q/t,\, q/t))}\; e^{-i\int_0^t ds\, A_t\,(\overleftrightarrow{\partial}_t G_{t-s} * j(v(q/t); s))}$$

Convergence to $\Psi^*_{out/in}(f)$:

- The coordinate space LSZ formulas are the result of a modified "coordinate space Dollard dynamics" (c.s.D.d. also applies to quantum Coulomb scattering)

$$U^D(t) = v^D(t)\, V^D(t)\, u_0(t)\, U_0(t)$$

$$v^D(t) = e^{i\,:\sigma\sigma:(\frac{e^2}{8\pi}\,\mathrm{sign}\,t\;\ln|t|\,t\,\frac{q-q'}{|q-q'|^3}\,(Q-Q'))}$$

$$V^D(t) = e^{-i\int dq\,(a(F_t^{D\varepsilon}(q/t))+h.c.)\,\sigma(q)}$$

$$e^{(\frac{i}{2}\int dq\,dq'\,L_t^\varepsilon(q/t,q'/t)\,\sigma(q)\sigma(q')+i\int dq\,\sigma(q)\,\delta E(q/t)\,\int_0^t ds\,e^{-2\varepsilon|s|})}$$

- The modified Dollard dynamics gives rise to the same Møller operators as strong limits (no adiabatic switching!)

- If a different, generic cutoff is used for the Dollard dynamics, then a smearing with $h_T(t)$ is necessary

q space LSZ formulas with corrections in asymptotic form:

$$\Psi^*_{out/in}(f) = \lim_t \int dQ\,dq\,\tilde{f}_{-t}(Q,q)\,\Psi^*_t(Q,q)$$

$$e^{i\rho_{out/in}\,(\chi_t(Q/t,q/t))}\;\; e^{-i\int_0^t ds\, B_{out/in}(j(v(q/t);s))}$$

No change for the asymptotic field algebra and dynamics

Interpretation in terms of gauge invariant bilinears: the argument of the exponential coincides, up to a correction which converges for $t \to \pm\infty$ (and neglecting a substitution $q \to Q$ in the formula for the A_0 component), with

$$-i \int_0^t ds\, A\,(j(v(q/t);s))$$

This implies convergence of

$$\int dQ\, dq\, \tilde{f}_{-t}(Q,q)$$

$$\Psi(g)\, e^{-i\int_0^t ds\, A\,(j(v(q/t);s)))}\, \Psi_t^*(Q,q)$$

i.e. the ordinary LSZ formula (also needing a $h_T(t)$ smearing in general), applied to one of the two variables of a gauge invariant bilinear

Similar (x space) gauge invariant fermion loops have been shown by Stapp to exhibit particle-like decay at large distances

Main conclusions on the LSZ procedure for charged fields:

- Realistic coordinate space modified LSZ formulas

- With the same results as the adiabatic procedure of Feynman diagrams with momentum space subtractions (of the Kulish-Faddev form)

- With an interpretation in terms of gauge invariant bilinears

- Asymptotic charged fields depend from a second space-time variable, "the origin of the string"

- Free particle spectrum appears at fixed "origin of the string"

- Asymptotic commutation relations between charged fields and e.m. fields are explicitely given by "the string"