An Algebraic Construction of Two-Dimensional Relativistic Quantum Theories

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Introduction

Basic observation on the $P(\varphi)_2$ model on the de Sitter space:
Fix a wedge W_1, let

$$\mathcal{A}_\circ(W_1) \doteq \text{algebra for } W_1 \text{ in the free theory,}$$
$$\Omega \doteq \text{interacting vacuum vector (which lies in Fock space),}$$
$$U_\circ(R) \doteq \text{free representation of the rotations } SO(2).$$

The triple $\big(\mathcal{A}_\circ(W_1), \Omega, U_\circ\big)$ fixes

- The representation of the *Lorentz group* (Boosts $\rightarrow$ modular unitary group)
- the entire *net* $\mathcal{O} \rightarrow \mathcal{A}(\mathcal{O})$ of the $P(\varphi)_2$ model.

Conjecture

1. Start with a triple $\big(\mathcal{A}_\circ(W_1), \Omega, U_\circ\big)$, where Ω is any *vector in Fock space satisfying certain specific conditions: Allows construction of a representation of the Lorentz group and of some interacting model.*
2. Such models can be carried over to $\mathbb{R}^{1+1}$ by considering the limit of infinite radius.

Geometry

- De Sitter space
$$dS \doteq \{x \in \mathbb{R}^{1+2} \mid x \cdot x = x_0^2 - x_1^2 - x_2^2 = -1\}$$
- Wedges: $W_1 \doteq \{x \in dS \mid x_1 > |x_0|\}$,
$$W = \Lambda W_1, \qquad \Lambda \in SO(1,2).$$
- Boosts and reflections:
$$\Lambda_1(t) \doteq \begin{pmatrix} \cosh t & \sinh t & 0 \\ \sinh t & \cosh t & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \Lambda_W(t) = \Lambda \Lambda_1(t) \Lambda^{-1}$$
$$\Theta_1 \doteq \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \Theta_W = \Lambda \Theta_1 \Lambda^{-1}$$
- We have
$$\Lambda_W(t) W = W, \quad t \in \mathbb{R},$$
$$\Theta_W W = W'.$$

- Rotations:

$$R_0(\alpha) \doteq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}, \quad \alpha \in [0, 2\pi) .$$

- "Euclidean sphere"

$$dS^{\mathbb{C}} \supset S^2 \doteq \{(ix_0, x_1, x_2) \in \mathbb{C}^{1+2} \mid x_0^2 + x_1^2 + x_2^2 = 1\}$$

$$S^2 \cap dS = S^1 \doteq \{(0, x_1, x_2) \in \mathbb{R}^{1+2} \mid x_1^2 + x_2^2 = 1\}$$

$$= \{(0, \cos\psi, \sin\psi) \mid \psi \in [-\pi, \pi)\}$$

$$\Lambda_1(i\theta) \cong R_1(\theta) \doteq \begin{pmatrix} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \\ 0 & 0 & 1 \end{pmatrix}, \quad \theta \in [0, 2\pi) .$$

(θ, ψ) are coordinates on S^2.

De Sitter models

$\mathcal{H} \doteq$ Hilbert space,

$U \doteq$ (anti-) unitary representation of $SO(1,2)$ in $\mathcal{H}$,

$\Omega \doteq U$- invariant vector in $\mathcal{H}$,

$\mathcal{A} : \mathcal{O} \to \mathcal{A}(\mathcal{O})$ family of v. Neumann algebras in $\mathcal{B}(\mathcal{H})$.

The quadruple $(\mathcal{H}, U, \Omega, \mathcal{A})$ satisfies the Haag-Kastler axioms if

Isotony:	$\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$ if $\mathcal{O}_1 \subset \mathcal{O}_2$
Locality:	$\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)'$ if $\mathcal{O}_1 \subset \mathcal{O}_2'$
Covariance:	$U(\Lambda)\mathcal{A}(\mathcal{O})U(\Lambda^{-1}) = \mathcal{A}(\mathcal{O})$
Cyclicity:	Ω is cyclic for each $\mathcal{A}(\mathcal{O})$.

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It satisfies modular covariance if for any wedge W the modular data J_W, Δ_W of $(\mathcal{A}(W), \Omega)$ are related to U by

$$\Delta_W^{it} = U\big(\Lambda_W(-2\pi t)\big), \quad J_W = U(\Theta_W).$$

$\Omega =$ KMS (2π): Geodesic KMS condtion of Borchers and Buchholz.

Free theory

Single particle space:
$\mathfrak{H} \doteq$ completion of $C^\infty(S^1)$ w.r.t. $\|h\|_{\mathfrak{H}}^2 \doteq \big(h, \frac{1}{2\omega} h\big)_{L^1(S^1)}$,
$\omega =$ pseudo diff. operator. Irrep. of $SO(1,2)$:

$$u(R)f \doteq R_* f \qquad R \in SO(2) \text{ or } R = P_1$$
$$u(T)f \doteq \bar{f}$$
$$u(\Lambda_W(t)) \doteq e^{-2\pi i t l_W}, \qquad l_{W_1} \doteq \omega \circ \widehat{\cos}.$$

$\mathcal{H} \doteq \Gamma(\mathfrak{H})$, $U_\circ(\Lambda) \doteq \Gamma(u(\Lambda))$.

$$\mathcal{A}_\circ(W) \doteq \text{Weyl}\{f : \; u(\Theta_W) e^{-\pi l_W} f = f\}$$
$$\mathcal{A}_\circ(\mathcal{O}) \doteq \bigcap_{W \supset \mathcal{O}} \mathcal{A}_\circ(W)$$

Modular unitary group of $(\mathcal{A}_\circ(W_1), \Omega_\circ)$:

$$\Delta_\circ^{it} = U_\circ(\Lambda_1(-2\pi t)) \doteq e^{it L_\circ}$$

Euclidean free theory:

$$\tilde{\mathfrak{H}} \doteq H^{-1}(S^2), \qquad \|f\|^2_{\tilde{\mathfrak{H}}} \doteq \big(f, (-\Delta + m^2)^{-1} f\big)_{L^2(S^2)}.$$ Contains $\mathfrak{H}$:

$$\mathfrak{H} \cong \{f \in \tilde{\mathfrak{H}} : \ \mathrm{supp} f \subset S^1\} \ \subset \tilde{\mathfrak{H}}$$

$$\tilde{\mathcal{H}} \doteq \Gamma(\tilde{\mathfrak{H}}) \xrightarrow{E_0} \mathcal{H} \qquad \tilde{\alpha}_R \doteq \mathrm{Ad}\,\Gamma(R_*), \quad R \in O(3)$$

$$e^{-\theta L_\circ} E_0 F \Omega_\circ = E_0 \tilde{\alpha}_{R_1(\theta)} F \Omega_\circ$$

Euclidean fields: For $f \in \tilde{\mathfrak{H}}$ real valued, $\phi(f) = \int d\Omega(x) f(x)\, \phi(x)$ generate abelian algebras.

Time-zero fields: $\phi(0,\psi)$ exist, $\psi \in S^1$. $\qquad \phi(\theta,\psi) = \tilde{\alpha}_{R_1(\theta)}\big(\phi(0,\psi)\big)$.

Wick monomials: $:e^{i\phi(f)}: \doteq e^{i\phi(f) - \|f\|^2/2}$.

$$:\phi^n:(f) = \int_{S^2} d\Omega(x) f(x) :\phi^n(x): \ \text{well-defined,}$$

$$= \int_0^{2\pi} d\theta \int_{S^1} d\psi \cos\psi\, f(\theta,\psi) \tilde{\alpha}_{R_1(\theta)}\big(:\phi^n:(0,\psi)\big).$$

Modular perturbation theory [Araki, Dyson, Schwinger]

$\mathcal{R} \doteq \mathcal{A}_\circ(W_1)$ of free model,

$\Omega_\circ \doteq$ free vacuum, $\Delta_\circ^{it} \doteq e^{itL_\circ} \doteq$ modular group for $(\mathcal{R}, \Omega_\circ)$.

Free dynamics : $\mathrm{Ad}\, e^{itL_\circ}$. $(\Omega_\circ, \cdot\, \Omega_\circ)$ is (-1)-KMS.

Given $V = V^* \in \mathcal{R}$, $H \doteq L_\circ + V$.

Interacting dynamics : $\mathrm{Ad}\, e^{itH}$. Has unique KMS (vector) state Ω.

Let $\Delta^{it} \doteq e^{itL} \doteq$ modular group for $(\mathcal{R}, \Omega)$,

$\Delta_{\mathrm{rel}} \doteq$ relative modular operator, $V_{\mathrm{rel}} \doteq$ relative Hamiltonian:

$$V_{\mathrm{rel}} \doteq -i \frac{d}{dt} \Delta_{\mathrm{rel}}^{it} \Delta_\circ^{-it}\Big|_{t=0},$$

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Theorem [Araki, Dyson, Schwinger,...]:

$$\Omega = e^{H/2}\Omega_\circ = \text{"}\, T \exp\Big\{ -\int_0^{1/2} ds\, e^{sL_\circ} V e^{-sL_\circ} \Big\}\text{"}\, \Omega_\circ \in \mathcal{P}(\mathcal{R}, \Omega_\circ),$$

$$\Delta_{\text{rel}}^{it} = e^{itH}, \qquad \mathrm{Ad}\, e^{itH} = \mathrm{Ad}\, e^{itL} \text{ on } \mathcal{R},$$

$$V_{\text{rel}} = V, \qquad L = H - JVJ.$$

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Note: 1-1 relation $\quad \Omega \leftrightarrow V$!

$P(\phi)$ model: An algebraic look.

$I_+ \doteq W_1 \cap S^1, \quad S_+ \doteq \{x \in dS : \; x^0 > 0\}$

$$V \doteq \int_{I_+} d\psi \cos\psi : \varphi^n(\psi) : \qquad \Omega \doteq E_0 \tilde{\Omega} \qquad \text{where}$$

$$\tilde{\Omega} \doteq \exp\Big\{ - \int_0^\pi d\theta \tilde{\alpha}_{R_1(\theta)}(V) \Big\} \Omega_\circ \equiv \exp\Big\{ - \int_{S_+} d\Omega(x) : \phi^n(x) : \Big\} \Omega_\circ$$

Lemma: Ω is the KMS state for Ad e^{itH}, with $H \doteq L_\circ + V$, and all relations from Araki's theorem hold.

Theorem

The modular unitary group Δ^{it} for $(\mathcal{A}_\circ(W_1), \Omega)$ and the free representation $U_\circ\big(SO(2)\big)$ generate a representation U of $SO_0(1,2)$.

Proof: Calculate $\Delta^{it}_{\rm rel}, \Delta^{it}$ explicitely:

$$e^{-\theta L} E_0 F \tilde{\Omega} = E_0 \tilde{\alpha}_{R_1(\theta)} F \tilde{\Omega}.$$

Use manifest $O(3)$-invariance of $\exp\big\{ - \int_{S^2} d\Omega(x) : \phi^n(x) : \big\}$
Construct "virtual representation" of $SO(3)$ [Fröhlich et al.]

Construction of the $P(\phi)$ model:

- for the wedge W_1, set $\mathcal{A}(W_1) \doteq \mathcal{A}_\circ(W_1)$;
- for an arbitrary wedge W, set

$$\mathcal{A}(W) \doteq U(\Lambda)\mathcal{A}\big(W_1\big)U(\Lambda)^{-1}, \qquad W = \Lambda W_1 \,;$$

- for a double cone $\mathcal{O} \subset dS$, set

$$\mathcal{A}(\mathcal{O}) = \bigcap_{\mathcal{O}\subset W} \mathcal{A}\big(W\big)\,.$$

Satisfies isotony, locality, covariance.

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Satisfies isotony, locality, covariance. What about cyclicity?
Consider double cones of the form I'', $I \subset S^1$.

Conjecture

$\mathcal{A}(I'') = \mathcal{A}_\circ(I'')$. *(Then* $(\mathcal{H}, U, \Omega, \mathcal{A})$ *satisfies all Haag-Kastler axioms.)*

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Proof: “Finite propagation speed” argument. $A \in \mathcal{A}_\circ(I'')$, $I \subset W_1$:

$$e^{itL_\circ} \underbrace{e^{itV} A e^{-itV}}_{\in \mathcal{A}_\circ(I'') \text{ since } V = V_I + V_{I^c},\, V_I \in \mathcal{A}_0(I''),\, V_{I^c} \in \mathcal{A}_0(I'')} e^{-itL_\circ} \in \mathcal{A}_\circ(\Lambda_1(t) I'') \subset \mathcal{A}_\circ(I''_t), \quad I \subset I_t \subset W_1$$

General construction of dS models.

The $P(\phi)$ model can be reconstructed entirely from (the free field and) the interacting vacuum vector Ω! (Since Ω fixes V.)
Relevant properties: Ω is

- cyclic for $\mathcal{A}_\circ(W_1)$
- in the natural positive cone $\mathcal{P}(\mathcal{A}_\circ(W_1), \Omega_\circ)$
- invariant under time-reflections and the group of rotations (for the free field).

Conjecture

Let Ω be any vector in Fock space satisfying the above conditions. Then a representation of the Lorentz group and a net of local algebras can be reconstructed from $\mathcal{A}_\circ(W_1)$, $U_\circ(SO(2))$ and Ω as in the $P(\phi)$ case.

From de Sitter to Minkowski Space

Geometry

$$dS^{(r)} \doteq \{x \in \mathbb{R}^{1+2} \mid x_0^2 - (x_1-1)^2 - x_2^2 = -r^2\} \quad \subset \mathbb{R}^{1+2}$$

$$x^{(r)}: \quad \mathbb{R}^{1+1} \to W_1^{(r)} \subset dS^{(r)} \quad \text{conformal equivalence}$$

$$x^{(\infty)} = \text{ canonical embedding } \mathbb{R}^{1+1} \hookrightarrow \mathbb{R} \times \{0\} \times \mathbb{R} \subset \mathbb{R}^{1+2}$$

$$E(1,1) \to \mathrm{Iso}(dS^{(r)}) \cong SO(1,2) \text{ group contraction}$$

$$g \mapsto g^{(r)}; \qquad g^{(\infty)} = g.$$

Unitary irreducible representations. Our reps $u \cong u_{mr}$, $m, r > 0$, of $SO_0(1,2)$ can all be realized on a common Hilbert space

$$\mathfrak{H} \cong L^2(\mathbb{R}, \frac{\mathrm{d}k}{2\sqrt{k^2+m^2}}) \oplus L^2(\mathbb{R}, \frac{\mathrm{d}k}{2\sqrt{k^2+m^2}}) \qquad \equiv \quad \mathfrak{H}_+ \oplus \mathfrak{H}_-$$

$$u_{mr}(g^{(r)}) \overset{r\to\infty}{\longrightarrow} v_m(g), \qquad \text{where} \qquad v_m = v_{m,+} \oplus v_{m,-}$$

is a reducible representation of $E_0(1,1)$ [Mickelsson& Niederle].

Free fields (dS to M)

For $\mathcal{O} \subset \mathbb{R}^{1+1}$, $f \in C_0^\infty(\mathcal{O})$ and $p \in \partial V_+$, let

$$\hat{f}^{(r)}(p) \doteq \int_{dS^{(r)}} \mathrm{d}\mu_{dS^{(r)}}\, f(t,q) \left(\frac{x^{(r)}(t,q)}{r} \cdot \frac{p}{m} \right)^{-\frac{1}{2}+imr} \subset \mathfrak{H}.$$

Lemma (Bros, Moschella)

Let $\mathcal{O}^{(r)} \doteq x^{(r)}\mathcal{O}$ *and let* $\mathcal{A}_\circ^{(r)}(\mathcal{O}^{(r)})$ *be the local algebra in the free* $dS^{(r)}$ *theory.*

$$(i) \qquad \mathcal{A}_\circ^{(r)}(\mathcal{O}^{(r)}) = \textit{Weyl}\,\{\hat{f}^{(r)} \mid f \in C_0^\infty(\mathcal{O})\}$$

$$(ii) \qquad \hat{f}^{(r)} \xrightarrow{r\to\infty} \hat{f} = \textit{ Fourier transform} \in \mathfrak{H}_+ \oplus \{0\} \subset \mathfrak{H}.$$

Recall that for the free Minkowski theory $\mathcal{A}_\circ(\mathcal{O}) = \text{Weyl}\,\{\hat{f} \mid f \in C_0^\infty(\mathcal{O})\}$. We therefore have:

$$\mathcal{A}_\circ(\mathcal{O}) = \left\{ \lim_{r\to\infty} V^{(r)}(\hat{f}^{(r)}) \mid f \in C_0^\infty(\mathcal{O}) \right\}''.$$

Interacting fields (dS to M)

We have for every $r > 0$ a dS model: $\Omega^{(r)}$, $U^{(r)}\big(SO(1,2)\big)$, $\mathcal{A}^{(r)}(\mathcal{O}^{(r)})$ acting in $\mathcal{H}$. Consider scaling algebras

$$\underline{\mathcal{A}}(\underline{\mathcal{O}}) \doteq \{\underline{A} : r \mapsto A^{(r)} \in \mathcal{A}^{(r)}(\mathcal{O}^{(r)})\}$$
$$\underline{\alpha_g}(\underline{A}) \doteq r \to U^{(r)}(g^{(r)})\ A^{(r)}\ U^{(r)}(g^{(r)})^{-1}, \quad g \in E(1,1)$$
$$\underline{\omega}_r(\underline{A}) \doteq \big(\Omega^{(r)}, A^{(r)}\Omega^{(r)}\big).$$

Let $\underline{\omega}_\infty$ be a w^* accumulation point of $(\underline{\omega}_r)_{r>0}$, with GNS triple $(\mathcal{H}, \pi, \Omega)$. Define for $\mathcal{O} \subset \mathbb{R}^{1+1}$

$$\mathcal{A}(\mathcal{O}) \doteq \pi\big(\underline{\mathcal{A}}(\underline{\mathcal{O}})\big), \alpha_g\pi(\underline{A}) \doteq \pi\big(\underline{\alpha_g}(\underline{A})\big)$$

Conjecture

The so-defined net satisfies the HK axioms, incl. the spectrum condition, except cyclicity for the local algebras.

Conjecture (Non-triviality)

Assume that for all $A \in \mathcal{A}_\circ(\mathcal{O}) \cong$ *Weyl* $\{V(\hat{f}) \mid \operatorname{supp} f \in \mathcal{O}\}$ *the limit*

$$\omega^\infty(A) \doteq \lim_{r\to 0} \big(\Omega^{(r)}, A\Omega^{(r)}\big)$$

exists and that, again for $\operatorname{supp} f \in \mathcal{O}$*,*

$$\omega^\infty\big(V(\hat{f})\big) = \lim_{r\to 0} \big(\Omega^{(r)}, V^{(r)}(\hat{f}^{(r)})\Omega^{(r)}\big)$$

Then $\mathcal{A}(\mathcal{O}_I) \cong \mathcal{A}_\circ(\mathcal{O}_I)$ *whenever* $\mathcal{O}_I$ *is based on the time-zero line, and* ω^∞ *is locally normal w.r.t. the Fock representation of the free field.*